PP38295

DANIEL SITARU - ROMANIA

In all triangles ABC holds:

$$1. \qquad \sum_{cyc} \frac{m_a w_a r_a}{bc} \geq \frac{s^2}{2R}$$

$$2. \qquad \sum_{cyc} \frac{m_a w_a r_a}{b+c} \geq \frac{sr(5s^2+r^2+4Rr)}{2(s^2+r^2+2Rr)}$$

Mihłaly Bencze

Solution by Daniel Sitaru.

$$1. \qquad \sum_{cyc} \frac{m_a w_b r_a}{bc} \geq \sum_{cyc} \frac{s(s-a)\cdot \frac{F}{s-a}}{bc} =$$

$$= sF \sum_{cyc} \frac{1}{bc} = \frac{s \cdot rs}{abc} \cdot \sum_{cyc} a =$$

$$= \frac{rs^2 \cdot 2s}{4Rrs} = \frac{s^2}{2R}$$

$$2. \qquad \sum_{cyc} \frac{m_a w_a r_a}{b+c} \geq \sum_{cyc} \frac{s(s-a)\cdot \frac{F}{s-a}}{b+c} =$$

$$= sF \cdot \sum_{cyc} \frac{1}{b+c} = s \cdot rs \cdot \frac{5s^2+r^2+4Rr}{2s(s^2+r^2+2Rr)} =$$

$$= \frac{sr(5s^2+r^2+4Rr)}{2(s^2+r^2+2Rr)}$$

Equality holds for $a = b = c$. □

Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania

Email address: dansitaru63@yahoo.com