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Prove without any software:

$$\int_{2-\sqrt{3}}^{1} e^{-x^2}\,dx < \frac{\pi}{6} \text{ and } \int_{1}^{2+\sqrt{3}} e^{-x^2}\,dx < \frac{\pi}{6}$$

Proposed by Neculai Stanciu-Romania

Solution by Ravi Prakash-New Delhi-India

Let $f(x) = (1+x^2)e^{-x^2} - 1; x \geq 0$, then

$$f'(x) = 2xe^{-x^2} - 2x(1+x^2)e^{-x^2} = 2xe^{-x^2}(1-1-x^2) = -2x^3e^{-x^2} < 0; \forall x > 0$$

$\Rightarrow f$ –is strictly decreasing on $[0,\infty) \Rightarrow f(x) < f(0), \forall x > 0$

$$\Rightarrow (1+x^2)e^{-x^2} < 1; \forall x \geq 0 \Rightarrow e^{-x^2} < \frac{1}{1+x^2}; \forall x > 0$$

$$\int_{2-\sqrt{3}}^{1} e^{-x^2}dx < \tan^{-1}x\Big|_{2-\sqrt{3}}^{1} = \frac{\pi}{4} - \frac{\pi}{12} = \frac{\pi}{6}$$

Also,

$$\int_{1}^{2+\sqrt{3}} e^{-x^2}\,dx = \tan^{-1}x\Big|_{1}^{2+\sqrt{3}} = \frac{5\pi}{12} - \frac{\pi}{4} < \frac{\pi}{6}$$