

## ABOUT NESBITT-IONESCU INEQUALITY

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If  $x, y, z > 0$  and  $u, v, w \geq 0$ , then:

$$\frac{x}{u+vy+wz} + \frac{y}{u+vz+wx} + \frac{z}{u+vx+wy} \geq \frac{3(x+y+z)}{3u+(v+w)(x+y+z)}; (*)$$

*Proof.* We have:

$$\begin{aligned} \sum_{cyc} \frac{x}{u+vy+wz} &= \sum_{cyc} \frac{x^2}{ux+vxy+wxz} \stackrel{Bergstrom}{\geq} \frac{(x+y+z)^2}{\sum_{cyc} (ux+vxy+wxz)} = \\ &= \frac{(x+y+z)^2}{u(x+y+z) + (v+w)(xy+yz+zx)} \geq \frac{(x+y+z)^2}{u(x+y+z) + (v+w)\frac{(x+y+z)^2}{3}} = \\ &= \frac{3(x+y+z)^2}{3u(x+y+z) + (v+w)(x+y+z)^2} = \frac{3(x+y+z)}{3u+(v+w)(x+y+z)} \end{aligned}$$

If  $u = 0$ , then  $v + w > 0$  and the inequality (\*) becomes

$$\frac{x}{vy+wz} + \frac{y}{vz+wx} + \frac{z}{vx+wy} \geq \frac{3}{v+w}; (**)$$

and if in (\*\*) we take  $v = w$ , we get Nesbitt-Ionescu's inequality, i.e.

$$\frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{x+y} \geq \frac{3}{2}; (N-I)$$

If in (\*) we take  $x + y + z = \frac{m}{n}$ ,  $m, n \in \mathbb{R}_+$ , then:

$$\frac{x}{u+vy+wz} + \frac{y}{u+vz+wx} + \frac{z}{u+vx+wy} \geq \frac{3\frac{m}{n}}{3u+(v+w)\frac{m}{n}} = \frac{3m}{3un+(v+w)m}; (***)$$

If in (\*\*\*) we take  $m = 3, n = 2$  and  $x + y + z = \frac{m}{n} = \frac{3}{2}$ , then:

$$\frac{x}{u+vy+wz} + \frac{y}{u+vz+wx} + \frac{z}{u+vx+wy} \geq \frac{3 \cdot 3}{3 \cdot 2u + 3(v+w)} = \frac{3}{2u+v+w}; (1)$$

If in (1) we take  $w = 0$ , we get:

$$\frac{x}{u+vy} + \frac{y}{u+vz} + \frac{z}{u+vx} \geq \frac{3}{2u+v}; (2)$$

If in (2) we take  $v = 1$ , it follows that:

$$\frac{x}{u+y} + \frac{y}{u+z} + \frac{z}{u+x} \geq \frac{3}{2u+1}; (M-C)$$

this is Marin Chirciu's inequality from [1]. page 35, and if in (M-C) we take  $u = 1$  we deduce that:

$$\frac{x}{1+y} + \frac{y}{1+z} + \frac{z}{1+x} \geq 1; (V.M.P.)$$

this is Vasile Mircea Popa's inequality.

REFERENCES

- [1]. Marin Chirciu, **ABOUT AN INEQUALITY** by Vasile Mircea Popa-**II-R.M.M.-37**, Summer Edition 2023.
- [2]. **ROMANIAN MATHEMATICAL MAGAZINE**-[www.ssmrmh.ro](http://www.ssmrmh.ro)
- [3]. **OCTOGON MATHEMATICAL MAGAZINE**