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If $f:[0,1]\to\mathbb{R}$ is continuous function such that

$$x^2+2\int_x^1 f(t)\,dt-1\ge 0,\forall x\in[0,1] \text{ then determine:}$$

$$\Omega=\min\left\{\int_0^1 f^2(t)\,dt\right\}$$

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Solution by Adrian Popa-Romania

$$\int_0^1 1^2\,dt+\int_0^1 f^2(t)\,dt\overset{CBS}{\ge}\left(\int_0^1 f(t)\,dt\right)^2$$

Hence,

$$\int_0^1 f^2(t)\,dt\ge\left(\int_0^1 f(t)\,dt\right)^2$$

$$x^2+2\int_x^1 f(t)\,dt-1\ge 0,\forall x\in[0,1],\text{then}$$

$$\int_x^1 f(t)\,dt\ge\frac{1-x^2}{2};\forall x\in[0,1]$$

$$\text{For } x=0, \text{we get: } \int_0^1 f(t)\,dt\ge\frac{1}{2}.$$

$$So, \int_0^1 f^2(t)\,dt \geq \frac{1}{4}. Therefore, \Omega = \min\left\{\int_0^1 f^2(t)\,dt\right\} = \frac{1}{4}$$