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If $0 < a \leq b$ then:

$$\left(\int_a^b e^{2x^2}\,dx\right)\left(\int_a^b e^{-x^2}\,dx\right) \geq (b-a)\int_a^b e^{x^2}\,dx$$

Proposed by Daniel Sitaru-Romania

Solution by Ruxandra Daniela Tonilă-Romania

If $a = b$ is nothing to prove.

If $a \neq b$ using Cauchy's inequality, we have:

$$\int_a^b \left(e^{x^2}\right)^2 dx \cdot \int_a^b dx \geq \left(\int_a^b e^{x^2}\,dx\right)^2$$

$$\int_a^b e^{x^2}\,dx \geq \frac{1}{b-a}\left(\int_a^b e^{x^2}\,dx\right)^2 \mid \cdot \int_a^b e^{-x^2}\,dx$$

$$\int_a^b e^{2x^2}\,dx \cdot \int_a^b e^{-x^2}\,dx \geq \frac{1}{b-a}\int_a^b e^{x^2}\,dx \cdot \int_a^b e^{x^2}\,dx \cdot \int_a^b e^{-x^2}\,dx$$

$$\int_a^b e^{2x^2}\,dx \cdot \int_a^b e^{-x^2}\,dx \geq \frac{1}{b-a}\int_a^b e^{x^2}\,dx \cdot \int_a^b \left(\sqrt{e^{x^2}}\right)^2 dx \cdot \int_a^b \left(\frac{1}{\sqrt{e^{x^2}}}\right)^2 dx \overset{CBS}{\geq}$$

$$\geq \frac{1}{b-a}\int_a^b e^{x^2}\,dx \cdot \left(\int_a^b \left(\sqrt{e^{x^2}} \cdot \frac{1}{\sqrt{e^{x^2}}}\right)dx\right)^2$$

Hence,

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$$\int_a^b e^{2x^2}\,dx \cdot \int_a^b e^{-x^2}\,dx \geq \frac{1}{b-a}\int_a^b e^{x^2}\,dx \cdot \left(\int_a^b dx\right)^2$$

Therefore,

$$\left(\int_a^b e^{2x^2}\,dx\right)\left(\int_a^b e^{-x^2}\,dx\right) \geq (b-a)\int_a^b e^{x^2}\,dx$$