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ROMANIAN MATHEMATICAL MAGAZINE

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Find:

$$\Omega=\int\left(\int e^x\cdot\frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}dx\right)dx$$

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$$\because\ erfi(z)=\frac{2}{\sqrt{\pi}}\int e^{z^2}\,dz$$

Now, we have:

$$\Omega=\int\left(\int e^x\cdot\frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}dx\right)dx=\int\left(\int e^x\cdot\frac{\sqrt{1+x}\left(\sqrt{1+x}+\sqrt{x}\right)}{\sqrt{x}+\sqrt{1+x}}dx\right)dx=$$

$$=\int\int e^x\sqrt{1+x}\,dx\,dx$$

$$\Omega_1=\int e^x\sqrt{1+x}\,dx\overset{t=\sqrt{1+x}}{=}\int e^{t^2-1}\cdot 2t^2\,dt=\frac{2}{e}\int t^2e^{t^2}\,dt\overset{IBP}{=}$$

$$=\frac{1}{e}\left(te^{t^2}-\int e^{t^2}\,dt\right)=\frac{1}{e}te^{t^2}-\frac{1}{e}\cdot\frac{\sqrt{\pi}}{2}erfi(t)+C$$

Hence,

$$\Omega=\underbrace{\frac{1}{e}\int\sqrt{1+x}e^{1+x}\,dx}_{\Omega_2}-\underbrace{\frac{\sqrt{\pi}}{2e}\int erfi\left(\sqrt{1+x}\right)dx}_{\Omega_3}+Cx$$

$$\Omega_3=\int erfi\left(\sqrt{1+x}\right)dx\overset{t=\sqrt{1+x}}{=}\int 2t\cdot erfi(t)\,dt=$$

$$=t^2\cdot erfi(t)-\frac{te^{t^2}}{\sqrt{\pi}}-\int t\cdot erfi(t)\,dt+\frac{1}{\sqrt{\pi}}\int e^{t^2}\,dt$$

Thus,

$$\frac{3}{2}\Omega_3=t^2\cdot erfi(t)-\frac{te^{t^2}}{\sqrt{\pi}}+\frac{1}{\sqrt{\pi}}\cdot\frac{\sqrt{\pi}}{2}\cdot erfi(t)$$

$$\Omega_3=\frac{2}{3}t^2\cdot erfi(t)-\frac{2te^{t^2}}{3\sqrt{\pi}}+\frac{1}{3}\cdot erfi(t)$$

$$\Omega_2=\int\sqrt{1+x}e^{1+x}\,dx\overset{\sqrt{1+x}=t}{=}\int te^{t^2}\cdot 2t\,dt=$$

$$=te^{t^2}-\frac{\sqrt{\pi}}{2}\cdot erfi(t)$$

Therefore,

$$\Omega=\frac{1}{e}\sqrt{1+x}e^{1+x}-\frac{\sqrt{\pi}}{2e}\cdot erfi\left(\sqrt{1+x}\right)-\frac{\sqrt{\pi}}{3e}(1+x)\cdot erfi\left(\sqrt{1+x}\right)+$$

$$+\frac{\sqrt{1+x}\cdot e^{1+x}}{3e}-\frac{erfi\left(\sqrt{1+x}\right)}{3}+Cx+C_1$$