R M M

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ABOUT A RMM INEQUALITY-XXII

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1) If $a,b,c>0$ such that $a+b+c=2$, then

$$\sum_{cyc}\frac{b^2+bc+c^2}{b+c}\ge 3$$

Proposed by Daniel Sitaru-Romania

Solution. Lemma. **2) If $x,y>0$ then**

$$\frac{x^2+xy+y^2}{x+y}\ge\frac{3(x+y)}{4}$$

Proof. We have:

$$\frac{x^2+xy+y^2}{x+y}\ge\frac{3(x+y)}{4}\Leftrightarrow(x-y)^2\ge 0,\text{equality holds for } x=y.$$

Let's get back to the main problem. Using Lemma, we have:

$$LHS=\sum_{cyc}\frac{b^2+bc+c^2}{b+c}\ge\sum_{cyc}\frac{3(b+c)}{4}=\frac{3}{2}\sum_{cyc}a=\frac{3}{2}\cdot 2=3$$

Equality holds for $a=b=c=\frac{2}{3}$.

Remark. The problem can be developed.

3) If $a,b,c\ge 0$ such that $a+b+c=2$ and $\lambda\le 2$ then

$$\sum_{cyc}\frac{b^2+\lambda bc+c^2}{b+c}\ge\lambda+2$$

Marin Chirciu

Solution. Lemma. **4) If $x,y>0$ and $\lambda\le 2$ then**

$$\frac{x^2+\lambda xy+y^2}{x+y}\ge\frac{\lambda+2}{4}(x+y)$$

Proof. We have:

$$\frac{x^2+\lambda xy+y^2}{x+y} \ge \frac{\lambda+2}{4}(x+y) \Leftrightarrow (2-\lambda)(x-y)^2 \ge 0$$

which is true from $(x-y)^2 \ge 0$ and $\lambda \le 2$. Equality holds for $x=y$.

Let's get back to the main problem. Using Lemma, we have:

$$LHS = \sum_{cyc}\frac{b^2+\lambda bc+c^2}{b+c} \ge \frac{\lambda+2}{4}\sum_{cyc}(b+c) = \frac{\lambda+2}{4}\cdot 2\sum_{cyc} a = \lambda+2$$

Equality holds for $a=b=c=\frac{2}{3}$.

Note.

For $\lambda=1$ we get the proposed problem by Daniel Sitaru in R.M.M. 10/2020.

REFERENCE:

ROMANIAN MATHEMATICAL MAGAZINE-www.ssmrmh.ro.