R M M

ROMANIAN MATHEMATICAL MAGAZINE

www.ssmrmh.ro

ABOUT A RMM INEQUALITY-XX

By Marin Chirciu-Romania

Edited by Florică Anastase-Romania

1) If $a, b, c > 0$ then

$$\sum_{cyc}\frac{a^4}{b^4}\cdot\sum_{cyc}\frac{a^3}{b^3}\ge\left(\sum_{cyc}\frac{a^2}{b^2}\right)^2$$

Proposed by Daniel Sitaru-Romania

Solution. Using AM-GM and CBS Inequality, we get:

$$\sum_{cyc}\frac{a^4}{b^4}\cdot\sum_{cyc}\frac{a^3}{b^3}\ge\sum_{cyc}\frac{a^4}{b^4}\cdot 3\sqrt[3]{\prod_{cyc}\frac{a^3}{b^3}}=\sum_{cyc}\frac{a^4}{b^4}\cdot 3=\sum_{cyc}\frac{a^4}{b^4}\cdot\sum_{cyc}1^2\ge$$

$$\ge\left(\sum_{cyc}\frac{a^2}{b^2}\cdot 1\right)^2=\left(\sum_{cyc}\frac{a^2}{b^2}\right)^2$$

Equality holds if and only if $a=b=c$.

Remark. The problem can be developed.

2) If $a, b, c > 0$ and $n \in \mathbb{N}^*$ then

$$\sum_{cyc}\frac{a^{2n}}{b^{2n}}\cdot\sum_{cyc}\frac{a^{2n-1}}{b^{2n-1}}\ge\left(\sum_{cyc}\frac{a^n}{b^n}\right)^2$$

Marin Chirciu

Solution. Using AM-GM and CBS Inequality, we get:

$$\sum_{cyc}\frac{a^{2n}}{b^{2n}}\cdot\sum_{cyc}\frac{a^{2n-1}}{b^{2n-1}}\ge\sum_{cyc}\frac{a^{2n}}{b^{2n}}\cdot 3\sqrt[3]{\prod_{cyc}\frac{a^{2n-1}}{b^{2n-1}}}=\sum_{cyc}\frac{a^{2n}}{b^{2n}}\cdot\sum_{cyc}1^2\ge$$

$$\ge\left(\sum_{cyc}\frac{a^n}{b^n}\cdot 1\right)^2=\left(\sum_{cyc}\frac{a^n}{b^n}\right)^2$$

Equality holds for $a=b=c$.

3) If $a,b,c>0$ and $n\in\mathbb{N}^*,k\in\mathbb{N}$ then

$$\sum_{cyc}\frac{a^{2n}}{b^{2n}}\cdot\sum_{cyc}\frac{a^k}{b^k}\ge\left(\sum_{cyc}\frac{a^n}{b^n}\right)^2$$

Marin Chirciu

Solution. Using AM-GM and CBS Inequality, we get:

$$\sum_{cyc}\frac{a^{2n}}{b^{2n}}\cdot\sum_{cyc}\frac{a^k}{b^k}\ge\sum_{cyc}\frac{a^{2n}}{b^{2n}}\cdot 3\sqrt[3]{\prod_{cyc}\frac{a^k}{b^k}}=\sum_{cyc}\frac{a^{2n}}{b^{2n}}\cdot 3=\sum_{cyc}\frac{a^{2n}}{b^{2n}}\cdot\sum_{cyc}1^2\ge$$

$$\ge\left(\sum_{cyc}\frac{a^n}{b^n}\cdot 1\right)^2=\left(\sum_{cyc}\frac{a^n}{b^n}\right)^2$$

Equality holds for $a=b=c$.

Note. For $n=2$ and $k=3$ we get the proposed problem by Daniel Sitaru-JP.348 from R.M.M.-24, Spring Edition 2022.

REFERENCE:

ROMANIAN MATHEMATICAL MAGAZINE-www.ssmrmh.ro