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ABOUT AN EQUATION BY JALIL HAJIMIR-I

By Marin Chirciu – Romania

1) Solve in $\mathbb{R}$:

$$2^{x^2-3x}+2^{x-x^2}=2^{1-x}$$

Jalil Hajimir – Canada

Solution: Using the means inequality we obtain:

$LHS=2^{x^2-3x}+2^{x-x^2}\geq 2\sqrt{2^{x^2-3x}\cdot 2^{x-x^2}}=2\cdot 2^{-x}=2^{1-x}=RHS$, with equality if and only if $2^{x^2-3x}=2^{x-x^2}\Leftrightarrow x^2-3x=x-x^2\Leftrightarrow x(x-2)=0\Leftrightarrow x\in\{0,2\}$.

The set of equation's solutions is $S=\{0,2\}$

Remark: The problem can be developed.

1.2 Let $\lambda\in\mathbb{R}$ fixed. Solve in $\mathbb{R}$:

$$2^{x^2-3\lambda x}+2^{\lambda x-x^2}=2^{1-\lambda x}$$

Marin Chirciu – Romania

Solution: Using the means inequality we obtain:

$LHS=2^{x^2-3\lambda x}+2^{\lambda x-x^2}\geq 2\sqrt{2^{x^2-3\lambda x}\cdot 2^{\lambda x-x^2}}=2\cdot 2^{-\lambda x}=2^{1-\lambda x}=RHS$, with equality if and only if $2^{x^2-3\lambda x}=2^{\lambda x-x^2}\Leftrightarrow x^2-3\lambda x=\lambda x-x^2\Leftrightarrow x(x-2\lambda)=0\Leftrightarrow x\in\{0,2\lambda\}$

The set of equation's solutions is $S=\{0,2\lambda\}$.

Note: For $\lambda=1$ we obtain the proposed problem by Jalil Hajimir, Canada, in Pascal Academy 10/2019.**Remark:** The problem can be developed.

1.3 Let $a,b\in\mathbb{R}$ fixed. Solve in $\mathbb{R}$:

$$2^{x^2-2ax}+2^{bx-x^2}=2^{1+(b-a)x}$$

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Solution: Using the means inequality we obtain:

$LHS=2^{x^2-2ax}+2^{2bx-x^2}\geq 2\sqrt{2^{x^2-2ax}\cdot 2^{2bx-x^2}}=2\cdot 2^{(b-a)x}=2^{1+(b-a)x}=RHS$, with equality if and only if $2^{x^2-2ax}=2^{2bx-x^2}\Leftrightarrow x^2-2ax=2bx-x^2\Leftrightarrow x(x-a-b)=0\Leftrightarrow$

$\Leftrightarrow x \in \{0, a+b\}$.The set of equation's solutions is $S = \{0, a+b\}$.

Note: For $a = \frac{3}{2}$ and $b = \frac{1}{2}$ we obtain the proposed problem by Jalil Hajimir, Canada, in Pascal Academy 10/2019. **Remark:** The problem can be developed.

2) Let $\lambda \in \mathbb{R}$ fixed. Solve in $\mathbb{R}$.

$$3^{2x^2-3\lambda x} + 3^{3\lambda x - x^2} + 3^{3\lambda x - x^2} = 3^{1+\lambda x}$$

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Solution: Using the means inequality we obtain:

$$LHS = 3^{2x^2-3\lambda x} + 3^{3\lambda x-x^2} + 3^{3\lambda x-x^2} \geq 3\sqrt[3]{3^{2x^2-3\lambda x} \cdot 3^{3\lambda x-x^2} \cdot 3^{3\lambda x-x^2}} = 3 \cdot 3^{\lambda x} =$$

$= 3^{1+\lambda x} = RHS$, with equality if and only if $3^{2x^2-3\lambda x} = 3^{3\lambda x-x^2} = 3^{3\lambda x-x^2} \Leftrightarrow$

$\Leftrightarrow 2x^2 - 3\lambda x = 3\lambda x - x^2 \Leftrightarrow 3x(x-2\lambda) = 0 \Leftrightarrow x \in \{0, 2\lambda\}$.

The set of equation's solutions is $S = \{0, 2\lambda\}$.

Remark: The problem can be developed.

3) Let $\lambda \in \mathbb{R}$ fixed. Solve in $\mathbb{R}$:

$$4^{3x^2-2\lambda x} + 4^{2\lambda x-x^2} + 4^{2\lambda x-x^2} + 4^{2\lambda x-x^2} = 4^{1+\lambda x}$$

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Solution: Using the means inequality we obtain:

$$LHS = 4^{3x^2-2\lambda x} + 4^{2\lambda x-x^2} + 4^{2\lambda x-x^2} + 4^{2\lambda x-x^2} \geq$$

$\geq 4\sqrt[4]{4^{3x^2-2\lambda x} \cdot 4^{2\lambda x-x^2} \cdot 4^{2\lambda x-x^2} \cdot 4^{2\lambda x-x^2}} = 4 \cdot 4^{\lambda x} = 4^{1+\lambda x} = RHS$, with equality if and only if $4^{3x^2-2\lambda x} = 4^{2\lambda x-x^2} = 4^{2\lambda x-x^2} = 4^{2\lambda x-x^2} \Leftrightarrow 3x^2 - 2\lambda x = 2\lambda x - x^2 \Leftrightarrow$

$\Leftrightarrow 4x(x-\lambda) = 0 \Leftrightarrow x \in \{0, \lambda\}$.

The set of equation's solution is $S = \{0, \lambda\}$.

Reference:

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