R M M

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ABOUT A RMM INEQUALITY-XIII

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1) If $a,b,c>0$ then:

$$\sum\frac{\sqrt{(a^2+b^2)(a^2+c^2)}}{a^2+bc}\geq 3$$

Daniel Sitaru – Romania

Solution: Using the inequality we have

$$(a^2+b^2)(a^2+c^2)\geq(a^2+bc)^2\ \ (1)\Leftrightarrow a^2(b-c)^2\geq 0$$

obviously with equality for $b=c$.

We obtain:

$$LHS=\sum\frac{\sqrt{(a^2+b^2)(a^2+c^2)}}{a^2+bc}\overset{(1)}{\geq}\sum\frac{a^2+bc}{a^2+bc}=3=RHS$$

Equality holds if and only if $a=b=c$.

Remark: The problem can be developed.

2) If $a,b,c>0$ and $\lambda\geq 0$ then:

$$\sum\frac{\sqrt{(a^2+\lambda b^2)(a^2+\lambda c^2)}}{a^2+\lambda bc}\geq 3$$

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Solution: For $\lambda=0$ we obtain the inequality $3=3$.

Using the inequality we have $(a^2+\lambda b^2)(a^2+\lambda c^2)\geq(a^2+\lambda bc)^2$ (1) $\Leftrightarrow\lambda a^2(b-c)^2\geq 0$

obviously with equality for $b=c$. We obtain:

$$RHS=\sum\frac{\sqrt{(a^2+\lambda b^2)(a^2+\lambda c^2)}}{a^2+\lambda bc}\overset{(1)}{\geq}\sum\frac{a^2+\lambda bc}{a^2+\lambda bc}=3=RHS$$

Equality holds if and only if $a=b=c$.

Note: For $\lambda=1$ we obtain the proposed problem by Daniel Sitaru in RMM 1/2021

Reference:

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