A SIMPLE PROOF FOR SCHREIBER'S INEQUALITY

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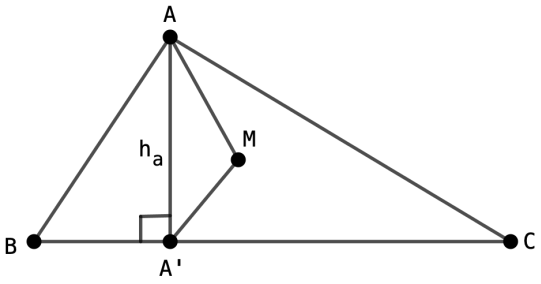
Abstract. In this paper it is given a simple proof for Schreiber's inequality in triangle published first time in 1935.

SCHREIBER'S INEQUALITY

If $M \in Int(\Delta ABC)$ then:

$$MA + MB + MC \geq 6r \tag{1}$$

Proof.



$$MA + MA' \geq h_a \Rightarrow MA' \geq h_a - MA \tag{2}$$

Analogous:

$$MB' \geq h_b - MB \tag{3}$$

$$MC' \geq h_c - MC \tag{4}$$

By Erdos-Mordell theorem:

$$MA + MB + MC \geq 2(MA' + MB' + MC') \geq$$

$$\overset{(2);(3);(4)}{\geq} 2(h_a + h_b + h_c) - 2(MA + MB + MC)$$

$$3(MA + MB + MC) \geq 2(h_a + h_b + h_c)$$

$$MA + MB + MC \geq \frac{2}{3}(h_a + h_b + h_c) = \frac{2}{3}\Big(\frac{2F}{a} + \frac{2F}{b} + \frac{2F}{c}\Big) =$$

$$= \frac{4F}{3} \cdot \frac{ab + bc + ca}{abc} = \frac{4F}{3} \cdot \frac{s^2 + r^2 + 4Rr}{4RF} =$$

$$= \frac{s^2 + r^2 + 4Rr}{3R} \geq 6r \Leftrightarrow$$

$$\Leftrightarrow s^2 + r^2 + 4Rr \geq 18Rr \Leftrightarrow$$

$$\Leftrightarrow s^2 \geq 14Rr - r^2$$

Key words and phrases. Schreiber, Erdos-Mordell.

By Gerretsen's inequality:

$$s^2 \geq 16Rr - 5r^2 \geq 14Rr - r^2 \Leftrightarrow$$

$$2Rr \geq 4r^2 \Leftrightarrow R \geq 2r \text{ (EULER)}$$

Equality holds for M - any center of an equilateral triangle. □

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