A SIMPLE PROOF FOR HUYGENS' INEQUALITY

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HUYGENS' INEQUALITY

If $0 < x < \frac{\pi}{2}$ then $2(\frac{\sin x}{x}) + \frac{\tan x}{x} > 3$.

Proof. Let be $f : (0, \frac{\pi}{2}) \to \mathbb{R}; f(x) = 2\sin x + \tan x - 3x$.

$$f'(x) = 2\cos x + \frac{1}{\cos^2 x} - 3$$

$$f''(x) = -2\sin x - \frac{(\cos^2 x)'}{(\cos^2 x)^2}$$

$$f''(x) = -2\sin x - \frac{2(\cos x)' \cdot \cos x}{\cos^4 x}$$

$$f''(x) = -2\sin x + \frac{2\sin x}{\cos^3 x}$$

$$f''(x) = -2\sin x\left(1 - \frac{1}{\cos^3 x}\right)$$

$$f''(x) = \frac{2\sin x(1 - \cos^3 x)}{\cos^3 x} > 0$$

$$f' \text{ increasing } \Rightarrow f'(x) > 0 > \lim_{\substack{x \to 0 \\ x > 0}} f'(x) = 0$$

$$f \text{ increasing } \Rightarrow f(x) > \lim_{\substack{x \to 0 \\ x > 0}} f(x) = 0 \Rightarrow f(x) > 0$$

$$2\sin x + \tan x - 3x > 0$$

$$2\sin x + \tan x > 3x$$

$$2\left(\frac{\sin x}{x}\right) + \frac{\tan x}{x} > 3$$

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