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ROMANIAN MATHEMATICAL MAGAZINE

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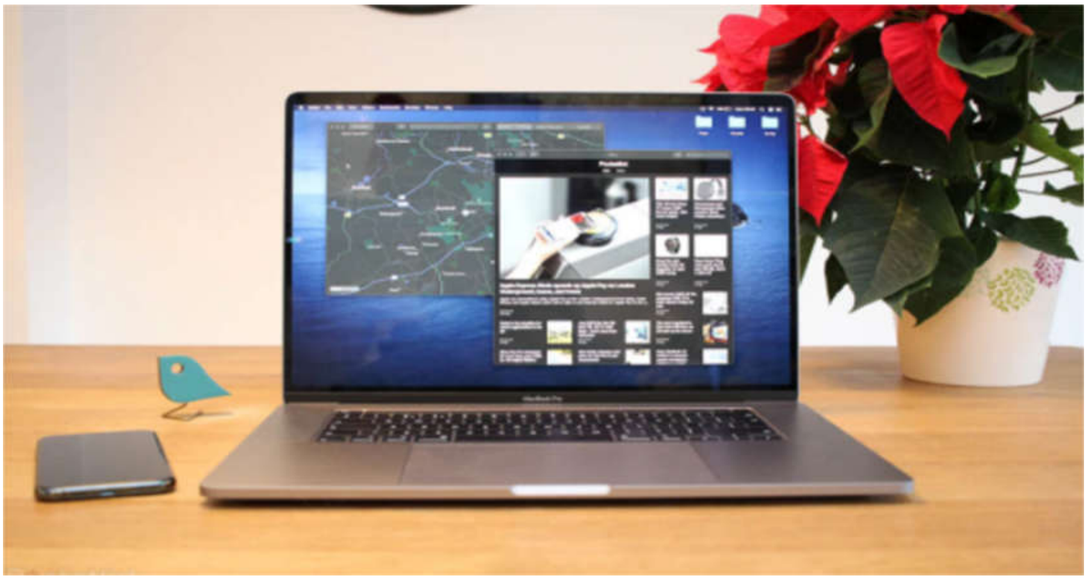

Prove that:

For $(m,n,p,q)\in N$

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\frac{(-1)^{m+n+p+q}}{n^2q^2((mp)^2+(nq)^2)}=\frac{\eta^4(2)}{2}$$

Proposed by Kaushik Mahanta-Assam-India

Solution by Asmat Qatea-Afghanistan

$$S=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\left(\frac{(-1)^{n+m+p+q}}{(nq)^2((mp)^2+(nq)^2)}\right)$$

$$S=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\left(\frac{(-1)^{n+m+p+q}}{(mp)^2}\left(\frac{1}{(nq)^2}-\frac{1}{((mp)^2+(nq)^2)}\right)\right)$$

$$S=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\left(\frac{(-1)^{n+m+p+q}}{(mpnq)^2}\right)-\underbrace{\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\left(\frac{(-1)^{n+m+p+q}}{(mp)^2((mp)^2+(nq)^2)}\right)}_{S}$$

$$2S=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{q=1}^{\infty}\left(\frac{(-1)^{n+m+p+q}}{(mpnq)^2}\right)$$

$$2S=\eta^4(2),\ S=\frac{1}{2}\eta^4(2)$$

Note:

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s}=\eta(s)$$