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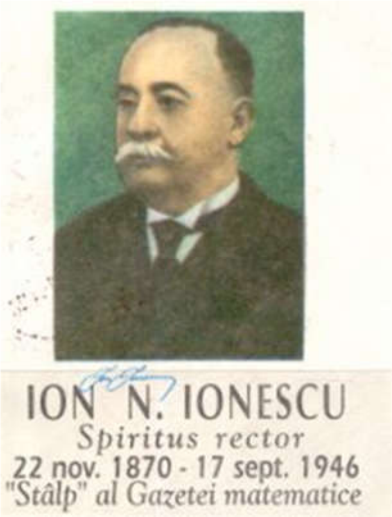


**Tribute to Ion Ionescu – who discovered with 22 years before Weitzenböck the inequality:**

$$a^2+b^2+c^2 \geq 4\sqrt{3}S$$

***By D.M.Bătinețu-Giurgiu,Neculai Stanciu-Romania***

**The authors of this proposed problem demonstrated in Romanian Mathematical Gazette , No. 1/2013, pp. 1-10, that the *Weitzenböck*'s inequality must be named the *Ionescu-Weitzenböck*'s inequality.**

**Our proof is based on: Romanian Mathematical Gazette, Vol. III (15 September 1897 – 15 August 1898), No. 2 , 15 October 1897, on page 52, *Ion Ionescu*, the founder of Romanian Mathematical Gazette, published the problem:*273. Prove that there is no triangle for which the inequality: $4S\sqrt{3} > a^2+b^2+c^2$ be satisfied. The solution of the problem 273, apperead in Romanian Mathematical Gazette, Vol. III (15 September 1897 – 15 August 1898), No. 12 , 15 August 1898, on pages 281, 282 and 283.**

**In the year 1919, *Roland Weitzenböck* published in Mathematische Zeitschrift, Vol. 5, No. 1-2, pp. 137-146 the article *Uber eine Ungleichung in der Dreiecksgeometrie*, where he proof that: In any triangle $ABC$, with usual notations holds the inequality: $a^2+b^2+c^2 \geq 4\sqrt{3}S$ .**

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**Proposed problem for RMM**

**If** $m \in R_+$**, then in all triangle** $ABC$**, with usual notations (i.e.** $R$ **=circumradius,** $r$ **=inradius, the lengths of the sides are** $a,b,c$ **and** $S$ **= the area of triangle** $ABC$**) the following inequality holds:**

$$\frac{a^{m+2}}{(b\cdot R+c\cdot r)^m}+\frac{b^{m+2}}{(c\cdot R+a\cdot r)^m}+\frac{c^{m+2}}{(a\cdot R+b\cdot r)^m}\geq\frac{4\sqrt{3}}{(R+r)^m}S.$$

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**Solution.** *We have:*

$$W=\sum_{cyc}\frac{a^{m+2}}{(b\cdot R+c\cdot r)^m}=\sum_{cyc}\frac{a^{2(m+1)}}{(abR+acr)^m}\geq 2^m\cdot\sum_{cyc}\frac{(a^2)^{m+1}}{((a^2+b^2)R+(a^2+c^2)r)^m},$$

*and applying the inequality of J. Radon we obtain:*

$$W\geq 2^m\cdot\frac{\left(\sum_{ciclic}a^2\right)^{m+1}}{(\sum_{ciclic}R(a^2+b^2)+\sum_{ciclic}r(a^2+c^2))^m}=2^m\cdot\frac{\left(\sum_{ciclic}a^2\right)^{m+1}}{2^m(R+r)^m\cdot(\sum_{ciclic}a^2)^m}=\frac{\sum_{ciclic}a^2}{(R+r)^m}.$$

*By Ionescu- Weitzenböck's inequality, i.e.* $\sum_{cyc}a^2\geq 4S\sqrt{3}$*, hence* $W\geq\frac{4S\sqrt{3}}{(R+r)^m}$*, and the proof is complete.*