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In ΔABC the following relationship holds :

$$\left(\sum m_a m_b\right)^2 + 2\left(\sum m_a^2\right)^2 \geq 3\sqrt{3}sr \left(\sum m_a\right)^2$$

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Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

We have :

$$\sum m_a m_b \stackrel{AM-GM}{\geq} 3\sqrt[3]{(m_a m_b m_c)^2} \stackrel{m_a m_b m_c \geq s^2 r}{\geq} 3\sqrt[3]{s^3 r^2 \cdot s} \stackrel{Mitrinovic}{\geq} 3s \cdot \sqrt[3]{3\sqrt{3}r^3} = 3\sqrt{3}sr$$

$$\rightarrow \sum m_a^2 \geq \sum m_a m_b \geq 3\sqrt{3}sr \quad (1)$$

$$\begin{aligned} \rightarrow \left(\sum m_a m_b\right)^2 + 2\left(\sum m_a^2\right)^2 &\stackrel{\sum m_a^2 \geq \sum m_a m_b}{\geq} 2\left(\sum m_a m_b\right)^2 + \left(\sum m_a^2\right)^2 \geq \\ &\stackrel{(1)}{\geq} 2 \cdot 3\sqrt{3}sr \sum m_a m_b + 3\sqrt{3}sr \sum m_a^2 = 3\sqrt{3}sr \left(\sum m_a\right)^2 \end{aligned}$$

Therefore,

$$\left(\sum m_a m_b\right)^2 + 2\left(\sum m_a^2\right)^2 \geq 3\sqrt{3}sr \left(\sum m_a\right)^2$$