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Find all numbers: $\Omega = \overline{abcd}$ such that:

$$\sqrt{(b+d)^2 - a^2 + 4c - 4} + 2(b^2 + d^2) + c^2 - 4a + 4 = 0$$

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Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

$$\sqrt{(b+d)^2 - a^2 + 4c - 4} + 2(b^2 + d^2) + c^2 - 4a + 4 \overset{(*)}{=} 0$$

(*) is defined when $(b+d)^2 - a^2 + 4c - 4 \ge 0 \Leftrightarrow (b+d)^2 \ge a^2 - 4c + 4$

We have: $2(b^2 + d^2) \ge (b+d)^2 \leftrightarrow (b-d)^2 \ge 0$

Which is true with equality holds when $b = d \rightarrow 2(b^2 + d^2) \ge a^2 - 4c + 4$

$$\rightarrow 2(b^2 + d^2) + c^2 - 4a + 4 \ge a^2 - 4c + 4 + c^2 - 4a + 4 = (a-2)^2 + (c-2)^2 \ge 0$$

$\rightarrow \sqrt{(b+d)^2 - a^2 + 4c - 4} + 2(b^2 + d^2) + c^2 - 4a + 4 \ge 0$, equality holds when:

$$\sqrt{(b+d)^2 - a^2 + 4c - 4} = 2(b^2 + d^2) + c^2 - 4a + 4 = 0$$

$\leftrightarrow b = d, a = c = 2$ and $(b+d)^2 = a^2 - 4c + 4 \rightarrow a = c = 2$ and $b = d = 0$

Therefore, $\Omega = \overline{abcd} = 2020.$