NEW LIMITS RELATED TO EULER-MASCHERONI AND IOACHIMESCU CONSTANTS

D.M. BĂTINEŢU-GIURGIU, MIHÁLY BENCZE, DANIEL SITARU, NECULAI STANCIU - ROMANIA

Abstract. In this paper we present some certain limits of real sequences.

Theorem 1.

$$\text{If } \gamma_n = -\ln n + \sum_{k=1}^{n} \frac{1}{k}, \lim_{n\to\infty} \gamma_n = \gamma \text{ (Euler-Mascheroni constant) and}$$

$$s_n = -2\sqrt{n} + \sum_{k=1}^{n} \frac{1}{\sqrt{k}}, \lim_{n\to\infty} s_n = s \text{ (Ioachimescu constant), then}$$

$$\lim_{n\to\infty} \frac{\gamma_n - \gamma}{(s_n - s)^2} = 2.$$

Proof.

$$\lim_{n\to\infty} \frac{\gamma_n - \gamma}{(s_n - s)^2} = \lim_{n\to\infty} \frac{n(\gamma_n - \gamma)}{(\sqrt{n}(s_n - s))^2} \tag{1}$$

$$\lim_{n\to\infty} n(\gamma_n - \gamma) = \lim_{n\to\infty} \frac{\gamma_n - \gamma}{\frac{1}{n}} \overset{\text{Cesaro-Stolz}}{=} \lim_{n\to\infty} \frac{\gamma_{n+1} - \gamma_n}{\frac{1}{n+1} - \frac{1}{n}} = \lim_{n\to\infty} \frac{\gamma_n - \gamma_{n+1}}{\frac{1}{n} - \frac{1}{n+1}} =$$

$$= \lim_{n\to\infty} \frac{-\ln n + \ln(n+1) - \frac{1}{n+1}}{\frac{1}{n(n+1)}} = \lim_{n\to\infty} \frac{\ln \frac{n+1}{n} - \frac{1}{n+1}}{\frac{1}{n(n+1)}} = \lim_{n\to\infty} \frac{\ln(1 + \frac{1}{n}) - \frac{1}{n+1}}{\frac{1}{n(n+1)}} =$$

$$= \lim_{\substack{x\to 0\\ x>0}} \frac{\ln(1+x) - \frac{x}{x+1}}{x \cdot \frac{x}{x+1}} = \lim_{\substack{x\to 0\\ x>0}} \frac{(x+1)\ln(x+1) - x}{x^2} \overset{\text{L'Hospital}}{=} \lim_{\substack{x\to 0\\ x>0}} \frac{\ln(x+1) + 1 - 1}{2x} =$$

$$= \lim_{\substack{x\to 0\\ x>0}} \frac{1}{2} \ln(1+x)^{\frac{1}{x}} = \frac{1}{2} \ln e = \frac{1}{2} \tag{2}$$

$$u = \lim_{n\to\infty} \sqrt{n}(s_n - s) = \lim_{n\to\infty} \frac{n(s_n - s)}{\sqrt{n}} \overset{\text{C-S}}{=} \lim_{n\to\infty} \frac{(n+1)(s_n - s) - n(s_n - s)}{\sqrt{n+1} - \sqrt{n}} =$$

$$= \lim_{n\to\infty} (\sqrt{n+1} + \sqrt{n})(n(s_{n+1} - s_n) + (s_{n+1} - s)) =$$

$$= 2 \lim_{n\to\infty} \sqrt{n} \cdot n \cdot (s_{n+1} - s_n) + 2 \lim_{n\to\infty} \sqrt{n+1}(s_{n+1} - s) =$$

$$= 2u + 2 \lim_{n\to\infty} \left(n\sqrt{n}\Big(\frac{1}{\sqrt{n+1}} - 2\sqrt{n+1} + 2\sqrt{n} \Big) \right) =$$

$$= 2u + 2 \lim_{n\to\infty} \left(\frac{n\sqrt{n}}{\sqrt{n+1}} (1 - 2(n+1) + 2\sqrt{n(n+1)}) \right) =$$

$$= 2u + 2 \lim_{n\to\infty} \frac{4n(n+1) - 4n^2 - 4n - 1}{2n + 1 + 2\sqrt{n(n+1)}} \cdot \frac{n\sqrt{n}}{\sqrt{n+1}} = 2u - \frac{1}{2} \Leftrightarrow u = \frac{1}{2} \tag{3}$$

From (1); (2) and (3) we get $\lim_{n\to\infty} \frac{\gamma_n - \gamma}{(s_n - s)^2} = 2$

□

Theorem 2.

If $\gamma_n = -\ln n + \sum_{k=1}^{n} \frac{1}{k}$, $\lim_{n\to\infty} \gamma_n = \gamma =$ Euler-Mascheroni constant, then

$$\lim_{n\to\infty} (\gamma_n - \gamma)\sqrt[n]{n!} = \frac{1}{2e}$$

Proof.

(1) Let $x_n = (\gamma_n - \gamma)\sqrt[n]{n!} = n(\gamma_n - \gamma) \cdot \frac{\sqrt[n]{n!}}{n}$

(2) It is well-known that $\lim_{n\to\infty} \frac{\sqrt[n]{n!}}{n} = \frac{1}{e}$

$$\lim_{n\to\infty} n(\gamma_n - \gamma) = \lim_{n\to\infty} \frac{\gamma_n - \gamma}{\frac{1}{n}} \overset{\text{Cesaro-Stolz}}{=} \lim_{n\to\infty} \frac{\gamma_{n+1} - \gamma_n}{\frac{1}{n+1} - \frac{1}{n}} = \lim_{n\to\infty} n(n+1)(\gamma_n - \gamma_{n+1}) =$$

$$= \lim_{n\to\infty} n(n+1)\left(\ln\frac{n+1}{n} - \frac{1}{n+1}\right) = \lim_{n\to\infty} n\left((n+1)\ln\left(1+\frac{1}{n}\right) - 1\right) =$$

$$= \lim_{\substack{x\to 0\\ x>0}} \frac{\frac{x+1}{x}\ln(1+x) - 1}{x} =$$

(3)

$$= \lim_{\substack{x\to 0\\ x>0}} \frac{(x+1)\ln(1+x) - x}{x^2} \overset{\text{L'Hospital}}{=} \lim_{\substack{x\to 0\\ x>0}} \frac{\ln(1+x)}{2x} = \frac{1}{2}\lim_{\substack{x\to 0\\ x>0}} \ln(1+x)^{\frac{1}{x}} = \frac{1}{2}\ln e = \frac{1}{2}$$

From (1), (2) and (3) we get $\lim_{n\to\infty} (\gamma_n - \gamma)\sqrt[n]{n!} = \lim_{n\to\infty} x_n = \frac{1}{2e}$.

□

Theorem 3.

If $\gamma_n = -\ln n + \sum_{k=1}^{n} \frac{1}{k}$, $\lim_{n\to\infty} \gamma_n = \gamma$, then $\lim_{n\to\infty} (\gamma_n - \gamma)\sqrt[n]{(2n-1)!!} = \frac{1}{e}$

Proof.

(1) Let $x_n = (\gamma_n - \gamma)\sqrt[n]{(2n-1)!!} = n(\gamma_n - \gamma) \cdot \frac{\sqrt[n]{(2n-1)!!}}{n}$

$$\lim_{n\to\infty} \frac{\sqrt[n]{(2n-1)!!}}{n} = \lim_{n\to\infty} \sqrt[n]{\frac{(2n-1)!!}{n^n}} \overset{\text{C-D'A}}{=} \lim_{n\to\infty} \left(\frac{(2n+1)!!}{(n+1)^{n+1}} \cdot \frac{n^n}{(2n-1)!!}\right) =$$

(2)
$$= \lim_{n\to\infty} \frac{2n+1}{n+1}\left(\frac{n}{n+1}\right)^n = \frac{2}{e}$$

$$\lim_{n\to\infty} n(\gamma_n - \gamma) = \lim_{n\to\infty} \frac{\gamma_n - \gamma}{\frac{1}{n}} \overset{\text{Cesaro-Stolz}}{=} \lim_{n\to\infty} \frac{\gamma_{n+1} - \gamma_n}{\frac{1}{n+1} - \frac{1}{n}} = \lim_{n\to\infty} n(n+1)(\gamma_n - \gamma_{n+1}) =$$

$$= \lim_{n\to\infty} n(n+1)\left(\ln\frac{n+1}{n} - \frac{1}{n+1}\right) = \lim_{n\to\infty} n\left((n+1)\ln\left(1+\frac{1}{n}\right) - 1\right) =$$

$$= \lim_{\substack{x\to 0\\ x>0}} \frac{\frac{x+1}{x}\ln(1+x)-1}{x} =$$

(3)

$$= \lim_{\substack{x\to 0\\ x>0}} \frac{(x+1)\ln(1+x)-x}{x^2} \overset{\text{L'Hospital}}{=} \lim_{\substack{x\to 0\\ x>0}} \frac{\ln(1+x)}{2x} = \frac{1}{2}\lim_{\substack{x\to 0\\ x>0}} \ln(1+x)^{\frac{1}{x}} = \frac{1}{2}\ln e = \frac{1}{2}$$

From (1), (2) and (3) we get $\lim_{n\to\infty}(\gamma_n-\gamma)\sqrt[n]{(2n-1)!!} = \lim_{n\to\infty} x_n = \frac{1}{e}$

□

Theorem 4.

$$\text{Find } \lim_{n\to\infty} \frac{\sqrt[n]{\sqrt{2!}\cdot\sqrt[3]{3!}\cdot\ldots\cdot\sqrt[n]{n!}}}{\sqrt[n+1]{(2n+1)!!}} = \frac{1}{2e}.$$

Proof.

(1)

$$\lim_{n\to\infty} \frac{\sqrt[n]{\sqrt{2!}\cdot\sqrt[3]{3!}\cdot\ldots\cdot\sqrt[n]{n!}}}{\sqrt[n+1]{(2n+1)!!}} = \lim_{n\to\infty}\left(\frac{\sqrt[n]{\sqrt{2!}\cdot\sqrt[3]{3!}\cdot\ldots\cdot\sqrt[n]{n!}}}{n}\cdot\frac{n+1}{\sqrt[n+1]{(2n+1)!!}}\cdot\frac{n}{n+1}\right)$$

$$\lim_{n\to\infty}\frac{\sqrt[n]{(2n-1)!!}}{n} = \lim_{n\to\infty}\sqrt[n]{\frac{(2n-1)!!}{n^n}} \overset{\text{C-D'A}}{=} \lim_{n\to\infty}\left(\frac{(2n+1)!!}{(n+1)^{n+1}}\cdot\frac{n^n}{(2n-1)!!}\right) =$$

(2)

$$= \lim_{n\to\infty}\frac{2n+1}{n+1}\left(\frac{n}{n+1}\right)^n = \frac{2}{e}$$

$$\lim_{n\to\infty} \frac{\sqrt[n]{\sqrt{2!}\cdot\sqrt[3]{3!}\cdot\ldots\cdot\sqrt[n]{n!}}}{n} = \lim_{n\to\infty}\sqrt[n]{\frac{\sqrt{2!}\cdot\sqrt[3]{3!}\cdot\ldots\cdot\sqrt[n]{n!}}{n^n}} =$$

$$= \lim_{n\to\infty}\frac{\sqrt{2!}\cdot\sqrt[3]{3!}\cdot\ldots\cdot\sqrt[n+1]{(n+1)!}}{(n+1)^{n+1}}\cdot\frac{n^n}{\sqrt{2!}\cdot\sqrt[3]{3!}\cdot\ldots\cdot\sqrt[n]{n!}} =$$

(3)

$$= \lim_{n\to\infty}\frac{\sqrt[n+1]{(n+1)!}}{n+1}\cdot\left(\frac{n}{n+1}\right)^n = \frac{1}{e}\lim_{n\to\infty}\frac{\sqrt[n]{n!}}{n} = \frac{1}{e^2}$$

From (1), (2) and (3) we get $\lim_{n\to\infty} \frac{\sqrt[n]{\sqrt{2!}\cdot\sqrt[3]{3!}\cdot\ldots\cdot\sqrt[n]{n!}}}{\sqrt[n+1]{(2n+1)!!}} = \frac{1}{e^2}\cdot\frac{e}{2}\cdot 1 = \frac{1}{2e}$

□

Theorem 5.

If $s_n = -2\sqrt{n} + \sum_{k+1}^{n}\frac{1}{\sqrt{k}}$, $\lim_{n\to\infty} s_n = s$ (Ioachimescu constant), then

$$\lim_{n\to\infty}(s_n - s)\sqrt[2n]{(2n-1)!!} = \frac{1}{\sqrt{2e}}$$

Proof.

(1)

$$\text{Let } x_n = (s_n - s)\sqrt[2n]{(2n-1)!!} = \sqrt{n}(s_n-s)\sqrt{\frac{\sqrt[n]{(2n-1)!!}}{n}}$$

$$\lim_{n\to\infty}\frac{\sqrt[n]{(2n-1)!!}}{n} = \lim_{n\to\infty}\sqrt[n]{\frac{(2n-1)!!}{n^n}} = \lim_{n\to\infty}\left(\frac{(2n+1)!!}{(n+1)^{n+1}}\cdot\frac{n^n}{(2n-1)!!}\right) =$$

(2) $$= \lim_{n\to\infty} \frac{2n+1}{n+1}\left(\frac{n}{n+1}\right)^n = \frac{2}{e}$$

$$\lim_{n\to\infty} \sqrt{n}(s_n - s) = \lim_{n\to\infty} \frac{s_n - s}{\frac{1}{\sqrt{n}}} \overset{\text{C-S}}{=} \lim_{n\to\infty} \frac{s_{n+1} - s_n}{\frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n(n+1)}}{\sqrt{n+1} - \sqrt{n}}(s_n - s_{n+1}) =$$

$$= \lim_{n\to\infty} \sqrt{n(n+1)}(\sqrt{n+1} + \sqrt{n})\Big(- 2\sqrt{n} - \frac{1}{\sqrt{n+1}} + 2\sqrt{n+1}\Big) =$$

(3)

$$= \lim_{n\to\infty} \sqrt{n}(\sqrt{n+1} + \sqrt{n})(2n+1 - 2\sqrt{n(n+1)}) = \lim_{n\to\infty} \frac{\sqrt{n}(\sqrt{n+1} + \sqrt{n})}{2n+1+2\sqrt{n(n+1)}} = \frac{1}{2}$$

From (1); (2) and (3) we get $\lim_{n\to\infty} (s_n - s) \sqrt[2n]{(2n-1)!!} = \lim_{n\to\infty} x_n = \frac{1}{2}\sqrt{\frac{2}{e}} = \frac{1}{\sqrt{2e}}$

□

Theorem 6.

If $s_n = -2\sqrt{n} + \sum_{k=1}^{n} \frac{1}{\sqrt{k}}$, $\lim_{n\to\infty} s_n = s$ (Ioachimescu constant), then

$$\lim_{n\to\infty} (s_n - s) \sqrt[2n]{n!} = \frac{1}{2\sqrt{e}}$$

Proof.

(1) Let $x_n = (s_n - s) \sqrt[2n]{n!} = \sqrt{n}(s_n - s)\sqrt{\frac{\sqrt[n]{n!}}{n}}$

(2) $$\lim_{n\to\infty} \frac{\sqrt[n]{n!}}{n} = \frac{1}{e}$$

$$\lim_{n\to\infty} \sqrt{n}(s_n - s) = \lim_{n\to\infty} \frac{s_n - s}{\frac{1}{\sqrt{n}}} \overset{\text{C-S}}{=} \lim_{n\to\infty} \frac{s_{n+1} - s_n}{\frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n(n+1)}}{\sqrt{n+1} - \sqrt{n}}(s_n - s_{n+1}) =$$

$$= \lim_{n\to\infty} \sqrt{n(n+1)}(\sqrt{n+1} + \sqrt{n})\Big(- 2\sqrt{n} - \frac{1}{\sqrt{n+1}} + 2\sqrt{n+1}\Big) =$$

(3)

$$= \lim_{n\to\infty} \sqrt{n}(\sqrt{n+1} + \sqrt{n})(2n+1 - 2\sqrt{n(n+1)} = \lim_{n\to\infty} \frac{\sqrt{n}(\sqrt{n+1} + \sqrt{n})}{2n+1+2\sqrt{n(n+1)}} = \frac{1}{2}$$

From (1), (2) and (3) we get $\lim_{n\to\infty} (s_n - s) \sqrt[2n]{n!} = \lim_{n\to\infty} x_n = \frac{1}{2}\sqrt{\frac{1}{e}} = \frac{1}{2\sqrt{e}}$

□

Theorem 7.

If $e_n = \left(1 + \frac{1}{n}\right)^n$, $\lim_{n\to\infty} e_n = e$ (Euler constant) and $s_n = -2\sqrt{n} + \sum_{k=1}^{n} \frac{1}{\sqrt{k}}$,

$\lim_{n\to\infty} s_n = s$ (Ioachimescu constant), then $\lim_{n\to\infty} \frac{e - e_n}{(s_n - s)^2} = 2e$.

Proof.

$$(1)\qquad \lim_{n\to\infty}\frac{e-e_n}{(s_n-s)^2}=\lim_{n\to\infty}\frac{n(e-e_n)}{(\sqrt{n}(s_n-s))^2}$$

$$\lim_{n\to\infty}\sqrt{n}(s_n-s)=\lim_{n\to\infty}\frac{s_n-s}{\frac{1}{\sqrt{n}}}\overset{\text{C-S}}{=}\lim_{n\to\infty}\frac{s_{n+1}-s_n}{\frac{1}{\sqrt{n+1}}-\frac{1}{\sqrt{n}}}=\lim_{n\to\infty}\frac{\sqrt{n(n+1)}}{\sqrt{n+1}-\sqrt{n}}(s_n-s_{n+1})=$$

$$=\lim_{n\to\infty}\sqrt{n(n+1)}(\sqrt{n+1}+\sqrt{n})\Big(-2\sqrt{n}-\frac{1}{\sqrt{n+1}}+2\sqrt{n+1}\Big)=$$

(2)

$$=\lim_{n\to\infty}\sqrt{n}(\sqrt{n+1}+\sqrt{n})(2n+1-2\sqrt{n(n+1)})=\lim_{n\to\infty}\frac{\sqrt{n}(\sqrt{n+1}+\sqrt{n})}{2n+1+2\sqrt{n(n+1)}}=\frac{1}{2}$$

$$(3)\qquad \text{It is well known that } \frac{e}{2n+2}<e-e_n<\frac{e}{2n+1}\Rightarrow\lim_{n\to\infty}n(e-e_n)=\frac{e}{2}$$

$$\text{From } (1),(2) \text{ and } (3) \text{ we get } \lim_{n\to\infty}\frac{e-e_n}{(s_n-s)^2}=2e$$

□

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Mathematics Department, National Economic College "Theodor Costescu", Drobeta Turnu - Severin, Romania

Email address: dansitaru63@yahoo.com