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If $A=\frac{sec^3x}{\sqrt{1+csc^2x}}+\frac{csc^3x}{\sqrt{1+sec^2x}}, x\in\left(0,\frac{\pi}{2}\right)$ then find $min\{A\}$.

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Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

$A=\frac{sec^3x}{\sqrt{1+csc^2x}}+\frac{csc^3x}{\sqrt{1+sec^2x}}, f(x)=\frac{1}{\sqrt{x}}$ –decreasing and convexe on $\left(0,\frac{\pi}{2}\right)$

$$\Rightarrow A=sec^2xf\left(cos^2x(1+csc^2x)\right)+csc^2xf\left(sin^2x(1+sec^2x)\right)\overset{JENSEN}{\geq}$$

$$\geq(sec^2x+csc^2x)f\left(\frac{1+csc^2x+1+sec^2x}{sec^2x+csc^2x}\right)=$$

$$=(sec^2x+csc^2x)f\left(1+\frac{2}{sec^2x+csc^2x}\right)$$

$sec^2x+csc^2x\geq\frac{4}{cos^2x+sin^2x}=4$ and $1+\frac{2}{sec^2x+csc^2x}\leq\frac{3}{2}\Rightarrow A\geq 4f\left(\frac{3}{2}\right)=\frac{4\sqrt{6}}{3}=min\{A\}$

Equality holds when $x=\frac{\pi}{4}$.

Note by editor:

Many thanks to Florică Anastase-Romania for typed solution.