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If $A = \frac{\tan^3 x}{\sqrt{1+\cot^2 x}} + \frac{\cot^3 x}{\sqrt{1+\tan^2 x}}, x \in \left(0, \frac{\pi}{2}\right)$ then find $\min\{A\}$.

Proposed by Marin Chirciu-Romania

Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

$A = \frac{\tan^3 x}{\sqrt{1+\cot^2 x}} + \frac{\cot^3 x}{\sqrt{1+\tan^2 x}}, f(x) = \frac{1}{\sqrt{x}}$ **is decreasing and convexe function on** $\left(0, \frac{\pi}{2}\right)$

$$\Rightarrow A = \tan^2 f\left(\cot^2 x(1+\cot^2 x)\right) + \cot^2 x f\left(\tan^2 x(1+\tan^2 x)\right) \overset{JENSEN}{\geq}$$

$$\geq (\tan^2 x + \cot^2 x) f\left(\frac{1+\cot^2 x + 1 + \tan^2 x}{\tan^2 x + \cot^2 x}\right)$$

$$\tan^2 x + \cot^2 x \overset{AM-GM}{\geq} 2 \Rightarrow 1 + \frac{2}{\tan^2 x + \cot^2 x} \leq 2 \Rightarrow A \geq 2f(2) = \sqrt{2} \Rightarrow$$

$\min\{A\} = \sqrt{2}$. **Equality holds when** $x = \frac{\pi}{4}$.

Note by editor:

Many thanks to Florică Anastase-Romania for typed solution.