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Prove without any software:

$$\int_{\frac{1}{16}}^{\frac{1}{8}} log\left(1+4x+8x^2+\frac{32x^4}{3}\right)dx<\frac{3}{128}$$

Proposed by Nikos Ntorvas-Greece

Solution by proposer

Let be the function $f(x)=36e^x-6x^3-18x^2, x\in\mathbb{R}; f''(x)=36(e^x-x-1)\geq 0$

Due to the well-known inequality $e^x\geq x+1, \forall x\in\mathbb{R}$ we have that f −is convex on $\mathbb{R}$.

The tangent for $A(0,f(0))$ is the $y=36x+36$. Because f −is convex we have that

$$f(x)\geq y, \forall x\in\mathbb{R}; (1)$$

So, we have that (1) holds for $x:=4x>0$

$$f(4x)\geq y\Rightarrow 36e^{4x}-6(4x)^3-18(4x)^2\geq 36(4x)+36\Leftrightarrow$$

$$4x\geq log\left(1+4x+8x^2+\frac{32x^4}{3}\right)$$

Therefore,

$$\int_{\frac{1}{16}}^{\frac{1}{8}} log\left(1+4x+8x^2+\frac{32x^4}{3}\right)dx<\frac{3}{128}$$

Note by editor:
Many thanks to Florică Anastase-Romania for typed solution.