R M M

ROMANIAN MATHEMATICAL MAGAZINE
www.ssmrmh.ro

In ΔABC the following relationship holds:

$$\sum_{cyc} \cos\frac{A}{2} \ge \frac{1}{2}\sum_{cyc}\frac{h_b + h_c}{a}$$

Proposed by Marin Chirciu-Romania

Solution by Marian Ursărescu-Romania

$$\frac{1}{2}\sum_{cyc}\frac{h_b + h_c}{a} = \frac{1}{2}\cdot 2F\sum_{cyc}\frac{\frac{1}{b}+\frac{1}{c}}{a} = sr\sum_{cyc}\frac{b+c}{abc} = \frac{sr}{abc}\sum_{cyc}(b+c) =$$

$$= \frac{sr}{4sRr}\cdot 4s = \frac{s}{R};$$

We must show that: $\sum_{cyc}\cos\frac{A}{2} \ge \frac{s}{R}$ (1)

But $\sum_{cyc}\cos\frac{A}{2} \ge 3\sqrt[3]{\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}} = 3\sqrt[3]{\frac{s}{4R}}$; (2)

From (1),(2) we must to prove that:

$$3\sqrt[3]{\frac{s}{4R}} \ge \frac{s}{R} \Leftrightarrow 2 + \frac{s}{4R} \ge \frac{s^3}{R^3} \Leftrightarrow \frac{27R^3}{4} \ge s^2 (MITRINOVIC)$$

Note by editor:

Many thanks to Florică Anastase-Romania for typed solution.