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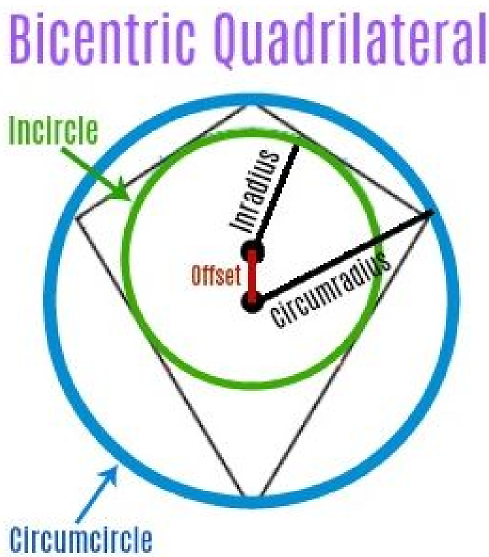

a, b, c, d −sides, −inradii in a bicentric quadrilateral. Prove that:

$$\frac{a^4}{b}+\frac{b^4}{c}+\frac{c^4}{d}+\frac{d^4}{a}\geq 32r^3$$

Proposed by Daniel Sitaru-Romania

Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

$$\frac{a^4}{b}+\frac{b^4}{c}+\frac{c^4}{d}+\frac{d^4}{a}\overset{BCS}{\geq}\frac{(\sum a)^4}{4^2\sum a}=\frac{(\sum a)^3}{16}\overset{(1)}{\geq}32r^3$$

$$(1)\Leftrightarrow \sum a\geq 8r$$

We know that in any bicentric quadrilateral $a+c=b+d$ and $r=\frac{\sqrt{abcd}}{a+c}=\frac{\sqrt{abcd}}{b+d}$

$$(1)\Leftrightarrow 2(a+c)\geq\frac{8\sqrt{abcd}}{b+d}\Leftrightarrow (a+c)(b+d)\geq 4\sqrt{abcd}$$

Which is clearly true from AM-GM: $a+c\geq 2\sqrt{ac}$ and $b+d\geq 2\sqrt{bd}$

Note by editor:

Many thanks to Florică Anastase-Romania for typed solutions.