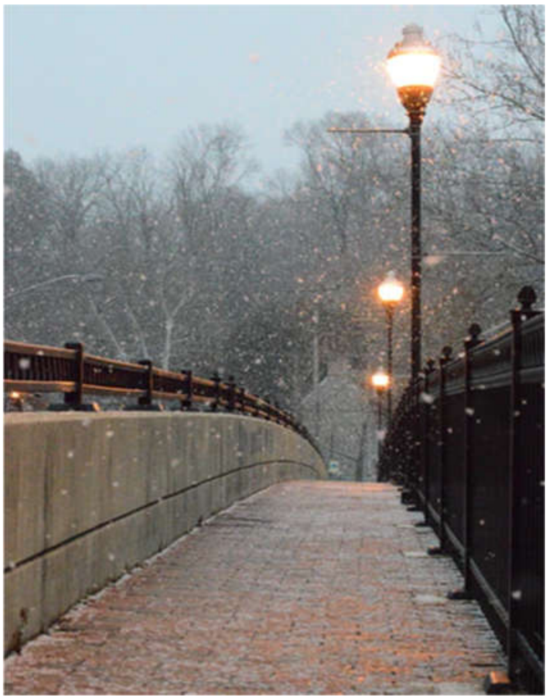
R M M

ROMANIAN MATHEMATICAL MAGAZINE
www.ssmrmh.ro



If $a,b,c>0$ and $a+b+c=3,\lambda\ge 0$ then:

$$\frac{a^6}{a^2+\lambda b}+\frac{b^6}{b^2+\lambda c}+\frac{c^6}{c^2+\lambda a}\ge\frac{3}{\lambda+1}$$

Proposed by Marin Chirciu-Romania

Solution by Michael Sterghiou-Greece

$$\frac{a^6}{a^2+\lambda b}+\frac{b^6}{b^2+\lambda c}+\frac{c^6}{c^2+\lambda a}\ge\frac{3}{\lambda+1};(1)$$

$$\frac{a^6}{a^2+\lambda b}+\frac{b^6}{b^2+\lambda c}+\frac{c^6}{c^2+\lambda a}\overset{Holder}{\ge}\frac{(a^2+b^2+c^2)^2}{3(\sum a^2)+3\lambda\sum a}\overset{(*)}{\ge}\frac{3}{\lambda+1}$$

As $a+b+c=3$ this reduces to $\frac{x^3}{3x+9\lambda}\ge\frac{3}{\lambda+1}$, with $x=\sum a^2$, which further reduces to

$(x-3)[(\lambda+1)x^2+3(\lambda+1)x+9\lambda]\ge 0$ which holds as

$$x=\sum_{cyc}a^2\ge\sum_{cyc}a=3.$$

Equality for $a=b=c=1$.

Note by editor:
Many thanks to Florică Anastase-Romania for typed solution.