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Solve in $\mathbb{C}$:

$$\begin{cases} (x+y)^2 = 5+xy \\ 9x^3 - 5x + 2xy^2 = 26y^3 + 5y - 2x^2y \end{cases}$$

Proposed by Carlos Paiva-Fortaleza-Brazil

Solution by Amir Sofi-Pristina-Kosovo

$$9x^3 - 5x + 2xy^2 = 26y^3 + 5y - 2x^2y \leftrightarrow$$

$$x^3 + y^3 - 5(x+y) + 2xy(x+y) + (2x)^3 - (3y)^3 = 0 \leftrightarrow$$

$$(+y)(x^2 - xy + y^2 + 2xy - 5) + (2x)^3 - (3y)^3 = 0 \leftrightarrow$$

$$(x+y)[(x+y)^3 - xy - 5] + (2x)^3 - (3y)^3 = 0 \leftrightarrow$$

$$(2x)^3 - (3y)^3 = 0 \leftrightarrow (2x-3y)(4x^2 + 6xy + 9y^2) = 0$$

$$4x^2 + 6xy + 9y^2 = 0 \leftrightarrow \frac{2x}{3y} + \frac{3y}{2x} + 1 = 0 \leftrightarrow \left(\frac{3y}{2x}\right)^2 + \frac{3y}{2x} + 1 = 0 \leftrightarrow$$

$$\frac{3x}{2y} = \frac{-1 \pm i\sqrt{3}}{2} \leftrightarrow y = \frac{-1 \pm i\sqrt{3}}{3}x$$

$$(x+y)^2 = 5 + xy$$

$$\left(x + \frac{-x + i\sqrt{3}x}{3}\right)^2 = 5 + x \cdot \frac{-x + i\sqrt{3}x}{3} \text{ v } \left(x + \frac{-x - i\sqrt{3}x}{3}\right)^2 = 5 + x \cdot \frac{-x - i\sqrt{3}x}{3}$$

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$$\left(\frac{2x+i\sqrt{3}x}{3}\right)^2=\frac{15-x^2+i\sqrt{3}x^2}{3} \text{ v } \left(\frac{2x-i\sqrt{3}x}{3}\right)^2=\frac{15-x^2-i\sqrt{3}x^2}{3}$$

$$4x^2+i\sqrt{3}x^2-45=0 \text{ v } 4x^2-i\sqrt{3}x^2-45=0$$

$$x=\pm\sqrt{\frac{45(4-i\sqrt{3})}{19}} \text{ v } x=\pm\sqrt{\frac{45(4+i\sqrt{3})}{19}}$$

$$(x,y)=\left(\pm\sqrt{\frac{45(4-i\sqrt{3})}{19}},\pm(-1\pm i\sqrt{3})\pm\sqrt{\frac{45(4-i\sqrt{3})}{19}}\right),$$

$$\left(\pm\sqrt{\frac{45(4+i\sqrt{3})}{19}},\pm(-1\pm i\sqrt{3})\pm\sqrt{\frac{45(4+i\sqrt{3})}{19}}\right)$$

Note by editor:
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