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Given x be the least prime divisor of the number $1\underbrace{000\ldots0}_{2018-times}1$ also

$$(2x)^{(2x)^{(2x)}} \equiv k(mod\ 100) \text{ then find } k.$$

Proposed by Rajeev Rastogi-India

Solution by Surjeet Singhania-India

$$1\underbrace{000\ldots0}_{2018-times}1 = 10^{2019} + 1 \equiv (\text{mod } 7) \to \text{x} = 7.$$

Denote $y = (2x)^{(2x)^{(2x)}} = 14^{14^{14}}$. We need to find $y(mod\ 100)$.

Since $4|y$, now we need to find $y(mod 25)$.

$y \equiv 14^{14^{14}}(mod\ 25)$. Since $\Phi(25) = 20 \to 14^{14} \equiv 6^{14}(mod\ 20)$

$14^{14} \equiv 16(mod\ 20) \to y \equiv 14^{16}(mod\ 25) \equiv 4^{8}(mod\ 25)(Euler\ th.)$. Hence,

$y \equiv 36(mod\ 25) \equiv 11(mod\ 25)$. Since $4|y \to y = 4k_1$.

$y = 4k_1 \equiv 11(mod\ 25) \to k_1 \equiv 9(mod 25) \to y = 100k_2 + 36.$

Hence, $y \equiv 14^{14^{14}} \equiv 36(mod\ 100) \to k = 36.$

Note by editor:
Many thanks to Florică Anastase-Romania for typed solution.