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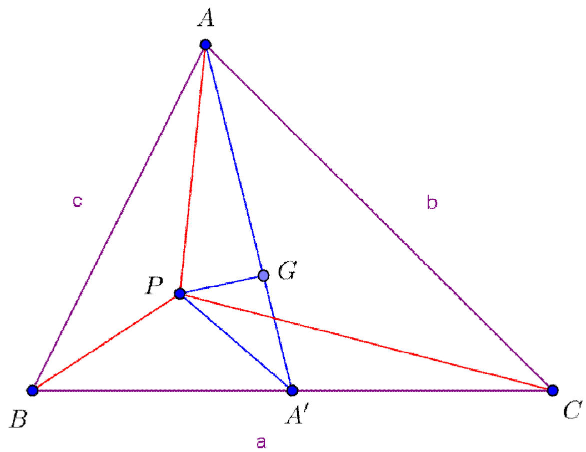

Let ΔDEF be the pedal triangle of $P \in Int(\Delta ABC)$ in ΔABC. Prove that:

$$AP^2 + BP^2 + CP^2 \geq \sqrt{3}F + \frac{4F^2}{9R^2}$$

When equality holds?

Proposed by Mehmet Şahin-Ankara-Turkey
Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

We prove that: $AP^2 + BP^2 + CP^2 \geq \frac{1}{3}\sum a^2 + 3PG^2$

Applying Stewart's theorem to $\Delta APA'$: $AA'(PG^2 + AG \cdot A'G) = A'P^2 \cdot AG + AP^2 \cdot A'G$

We have $AG = \frac{2}{3}AA', A'G = \frac{1}{3}AA'$, then

$$PG^2 + \frac{2}{9}A'A^2 = \frac{2}{3}A'P^2 + \frac{1}{3}AP^2$$

$A'P$ –is the medians in $\Delta BPC \Rightarrow A'P^2 = \frac{BP^2+CP^2}{2} - \frac{a^2}{4}, A'A^2 = \frac{b^2+c^2}{2} - \frac{a^2}{4}$

$$AP^2 + BP^2 + CP^2 = \frac{1}{3}\sum a^2 + 3PG^2 \geq \frac{1}{3}\sum a^2 \geq \frac{4\sqrt{3}}{3}F =$$

$$= \sqrt{3}F + \frac{\sqrt{3}}{3}\frac{F^2}{sr}$$

Equality holds if and only if $P = G$.

We have: $r \leq \frac{R}{2}, s \leq \frac{3\sqrt{3}}{2}R \Rightarrow \frac{F^2}{sr} \geq \frac{4F^2}{3\sqrt{3}R^2} \Rightarrow$

$$AP^2 + BP^2 + CP^2 \geq \sqrt{3}F + \frac{4F^2}{9R^2}$$

Equality holds if and only if triangle is equilateral.

Note by editor:

Many thanks to Florică Anastase-Romania for typed solution.