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In ΔABC the following relationship holds:

$$\frac{m_b m_c}{m_a}+\frac{m_c m_a}{m_b}+\frac{m_a m_b}{m_c}\le\frac{9R^2}{4r}$$

Proposed by Eldeniz Hesenov-Georgia

Solution by Tran Hong-Dong Thap-Vietnam

$$\frac{m_b m_c}{m_a}+\frac{m_c m_a}{m_b}+\frac{m_a m_b}{m_c}=m_a m_b m_c\left(\frac{1}{m_a^2}+\frac{1}{m_b^2}+\frac{1}{m_c^2}\right)\overset{m_a\ge\sqrt{s(s-a)}}{\le}$$

$$\le m_a m_b m_c\left(\frac{1}{s(s-a)}+\frac{1}{s(s-b)}+\frac{1}{s(s-c)}\right)=$$

$$=m_a m_b m_c\cdot\frac{(s-b)(s-c)+(s-c)(s-a)+(s-a)(s-b)}{s(s-a)(s-b)(s-c)}=$$

$$=m_a m_b m_c\cdot\frac{r(4R+r)}{s^2r^2}=m_a m_b m_c\cdot\frac{4R+r}{rs^2}\overset{m_a m_b m_c\le\frac{Rs^2}{2}}{\le}\frac{Rs^2}{2}\cdot\frac{4R+r}{rs^2}=$$

$$=\frac{R(4R+r)}{2r}\overset{r\le\frac{R}{2}}{\le}\frac{R\cdot\frac{9R}{2}}{2r}=\frac{9R^2}{4r}$$

Note by editor:

Many thanks to Florică Anastase-Romania for typed solution.