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$$\sqrt{(x+y)(4-z)}+\sqrt{(y+z)(2-t)}+\sqrt{(z+t)(4-x)}+\sqrt{(t+x)(2-y)}$$

$$=\sqrt{\frac{5s^2-24S+144}{2}};s=x+y+z+t$$

Find: $\Omega=\overline{xyzt}$

Proposed by George Florin Şerban-Romania

Solution 1 by proposer, Solution 2 by Michael Sterghiou-Greece

Solution 1 by proposer

$$\sqrt{(x+y)(4-z)}\overset{AM-GM}{\leq}\frac{x+y+4-z}{2},\sqrt{(y+z)(2-t)}\overset{AM-GM}{\leq}\frac{y+z+2-t}{2},$$

$$\sqrt{(z+t)(4-x)}\overset{AM-GM}{\leq}\frac{z+t+4-x}{2},\sqrt{(t+x)(2-y)}\overset{AM-GM}{\leq}\frac{t+x+2-y}{2}$$

Thus,

$$\sqrt{\frac{5s^2-24S+144}{2}}\leq\frac{s+12}{2}$$

$$\sqrt{\frac{(2s)^2+(12-s)^2}{2}}\leq\frac{2s+(12-s)}{2}\leq\sqrt{\frac{(2s)^2+(12-s)^2}{2}}$$

Applying AM-GM inequality, equality holds when $2s=12-s,s=4\rightarrow$

$$x+y=4-z, y+z=2-t, z+t=4-x, t+x=2-y$$

$$x+y+z+t=4, x+y+z=4 \to t=0, x+z+t=4 \to y=0, x=2, \overline{xyzt}=2020.$$

Solution 2 by Michael Sterghiou-Greece

$$\sqrt{(x+y)(4-z)}+\sqrt{(y+z)(2-t)}+\sqrt{(z+t)(4-x)}+\sqrt{(t+x)(2-y)}$$

$$=\sqrt{\frac{5s^2-24S+144}{2}}; (1)$$

$x,y,z,t\in\mathbb{N}$. **By AM-GM,** $LHS_{(1)}\le\frac{s+12}{2}$ **and** $RHS_{(1)}\le\frac{s+12}{2}\to\frac{9}{4}(s-4)^2\le 0\to s=4$.

We see that $(x,y,z,t)=(2,0,2,0)$ **is the only solution giving** $LHS_{(1)}=8=RHS_{(1)}$ **with** $s=4$. **Therefor,** $\Omega=\overline{xyzt}=2020$.

Note by editor:

Many thanks to Florică Anastase-Romania for typed solutions.