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ABOUT PROBLEM W6-JOSZEF WILDT

By Marin Chirciu – Romania

1) Compute:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{(1+\ln x)\cos x + x\ln x\sin x}{\cos^2 x + x^2\ln^2 x}dx$$

Proposed by D.M. Bătinețu-Giurgiu and Neculai Stanciu – Romania

Solution

$$\text{We have } \left(\frac{x\ln x}{\cos x}\right)' = \frac{(1+\ln x)\cos x + x\ln x\sin x}{\cos^2 x}$$

$$\text{We obtain } \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{(1+\ln x)\cos x + x\ln x\sin x}{\cos^2 x + x^2\ln^2 x}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\left(\frac{x\ln x}{\cos x}\right)'}{1+\left(\frac{x\ln x}{\cos x}\right)^2}dx = \arctan\left(\frac{x\ln x}{\cos x}\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} =$$

$$= \arctan\left(\frac{\frac{\pi}{4}\ln\frac{\pi}{4}}{\cos\frac{\pi}{4}}\right) - \arctan\left(\frac{\frac{\pi}{6}\ln\frac{\pi}{6}}{\cos\frac{\pi}{6}}\right) = \arctan\left(\frac{\pi\sqrt{2}}{4}\ln\frac{\pi}{4}\right) - \arctan\left(\frac{\pi\sqrt{3}}{9}\ln\frac{\pi}{6}\right)$$

Remark. *In the same way:*

2) Find:

$$\int_1^e \frac{e^x(1+\ln x - \ln^2 x)}{e^{2x} + x^2\ln^2 x}dx$$

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Solution

$$\text{We have } \left(\frac{x\ln x}{e^x}\right)' = \frac{(1+\ln x)e^x - x\ln x\cdot e^x}{e^{2x}} = \frac{e^x(1+\ln x - x\ln x)}{e^{2x}}$$

$$\text{We obtain } \int_1^e \frac{e^x(1+\ln x-\ln^2 x)}{e^{2x}+x^2\ln^2 x}dx = \int_1^e \frac{\left(\frac{x\ln x}{e^x}\right)'}{1+\left(\frac{x\ln x}{e^x}\right)^2}dx = \arctan\left(\frac{x\ln x}{e^x}\right)\Big|_1^e =$$

$$= \arctan\left(\frac{e\ln e}{e^e}\right) - \arctan\left(\frac{1\ln 1}{e}\right) = \arctan\left(\frac{e}{e^e}\right) = \arctan e^{1-e}$$

3) Find:

$$\int \frac{e^x(1+\ln x-\ln^2 x)}{e^{2x}+x^2\ln^2 x}dx, x\in(0,\infty)$$

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Solution *We have* $\left(\frac{x\ln x}{e^x}\right)' = \frac{(1+\ln x)e^x - x\ln x\cdot e^x}{e^{2x}} = \frac{e^x(1+\ln x-x\ln x)}{e^{2x}}$

We obtain $\int \frac{e^x(1+\ln x-\ln^2 x)}{e^{2x}+x^2\ln^2 x}dx = \int \frac{\left(\frac{x\ln x}{e^x}\right)'}{1+\left(\frac{x\ln x}{e^x}\right)^2}dx = \arctan\left(\frac{x\ln x}{e^x}\right)+C$

4) Find:

$$\int_1^e \frac{x+1+\ln x}{(x+1)^2+x^2\ln^2 x}dx$$

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Solution *We have* $\left(\frac{x\ln x}{x+1}\right)' = \frac{(1+\ln x)(x+1)-x\ln x}{(x+1)^2} = \frac{x+1+\ln x}{(x+1)^2+x^2\ln^2 x}$

We obtain $\int_1^e \frac{x+1+\ln x}{(x+1)^2+x^2\ln^2 x}dx = \int_1^e \frac{\left(\frac{x\ln x}{x+1}\right)'}{1+\left(\frac{x\ln x}{x+1}\right)^2}dx = \arctan\left(\frac{x\ln x}{x+1}\right)\Big|_1^e =$

$$= \arctan\left(\frac{e\ln e}{e+1}\right) - \arctan\left(\frac{1\ln 1}{2}\right) = \arctan\left(\frac{e}{e+1}\right)$$

5) Find:

$$\int \frac{x+1+\ln x}{(x+1)^2+x^2\ln^2 x}dx, x\in(0,\infty)$$

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Solution *We have* $\left(\frac{x\ln x}{x+1}\right)' = \frac{(1+\ln x)(x+1)-x\ln x}{(x+1)^2} = \frac{x+1+\ln x}{(x+1)^2+x^2\ln^2 x}$

We obtain $\int \frac{x+1+\ln x}{(x+1)^2+x^2\ln^2 x}dx = \int \frac{\left(\frac{x\ln x}{x+1}\right)'}{1+\left(\frac{x\ln x}{x+1}\right)^2}dx = \arctan\left(\frac{x\ln x}{x+1}\right)+C$

Reference:

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