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TRAIAN LALESCU LIMIT - 120 YEARS D.M. BĂTINEŢU - GIURGIU - 85 YEARS

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Article dedicated to D.M. Bătineţu - Giurgiu

Abstract. Next, we present some theorems with Lalescu type and D.M. Bătineţu - Giurgiu's limits

1) Traian Lalescu's limit (G.M., Vol. VI, 1900-1901, problem 579, p. 148).

$$\lim_{n\to\infty}\left(\sqrt[n+1]{(n+1)!}-\sqrt[n]{n!}\right)=\frac{1}{e}$$

2) D.M. Bătineţu - Giurgiu's limit (G.M., Vol XCIV, 1989, problem C:890, p.139).

$$\lim_{n\to\infty}\left(\frac{(n+1)^2}{\sqrt[n+1]{n!}}-\frac{n^2}{\sqrt[n]{n!}}\right)=e$$

3) Romeo T. Ianculescu's limit (G.M., Vol. XIX, 1913-1914, problem 2042, p. 160).

$$\lim_{n\to\infty}\left((n+1)\sqrt[n+1]{(n+1)}-n\sqrt[n]{n}\right)=1$$

4) D.M. Bătineţu - Giurgiu, Neculai Stanciu, SSM 4/2016
If $(2n-1)!!=1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1),\forall n\in\mathbb{N}^*$, then:

$$\lim_{n\to\infty}\left(\frac{\sqrt[n+1]{(n+1)!(2n+1)!!}}{n+1}-\frac{\sqrt[n]{n!(2n-1)!!}}{n}\right)=\frac{2}{e^2}$$

5) D.M. Bătineţu - Giurgiu, Neculai Stanciu, SSM 4/2012

If $(a_n)_{n\geq 1}, a_n\in\mathbb{R}_+^*,\forall n\in\mathbb{N}^*$ such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{n^2\cdot a_n}=b\in\mathbb{R}_+^*$, then

$$\lim_{n\to\infty}\left(\frac{\sqrt[n+1]{a_{n+1}}}{n+1}-\frac{\sqrt[n]{a_n}}{n}\right)=\frac{b}{e^2}$$

6) D.M. Bătineţu - Giurgiu, Neculai Stanciu, MP 4/2010-2011, P 1/2012.

If $(a_n)_{n\geq 1},(b_n)_{n\geq 1}, a_n, b_n\in\mathbb{R}_+^*,\forall n\in\mathbb{N}^*$ such that

$$\lim_{n\to\infty}\frac{a_{n+1}}{n^2\cdot a_n}=a\in\mathbb{R}_+^*\text{ and }\lim_{n\to\infty}\frac{b_{n+1}}{n^3\cdot b_n}=b\in\mathbb{R}_+^*\text{ then}$$

$$\lim_{n\to\infty}\left(\sqrt[n+1]{\frac{b_{n+1}}{a_{n+1}}}-\sqrt[n]{\frac{b_n}{a_n}}\right)=\frac{b}{ae}$$

7) D.M. Bătineţu - Giurgiu, Neculai Stanciu, CM 2/2012.

If $e_n=\left(1+\frac{1}{n}\right)^n$ and $\gamma_n=-\ln n+\sum\limits_{k=1}^{n}\frac{1}{k}$, then

$$\lim_{n\to\infty}\left(\frac{(n+1)^2}{\sqrt[n+1]{(2n+1)!!\gamma_n}}-\frac{n^2}{\sqrt[n]{(2n-1)!!e_n}}\right)=\frac{e}{2}(2-\ln\gamma)$$

8) D.M. Bătineţu - Giurgiu, Neculai Stanciu, LG 3/2013.

If $f:\mathbb{R}_+^*\to\mathbb{R}_+^*$ with $\lim\limits_{x\to\infty}\frac{f(x)}{x}=c\in\mathbb{R}_+^*$ and $(a_n)_{n\geq 1}$ a positive sequence such that $\lim\limits_{n\to\infty}(a_{n+1}-a_n)=a\in\mathbb{R}_+^*$, then

$$\lim_{n\to\infty}\left(\frac{(n+1)^2}{\sqrt[n+1]{f(a_1)f(a_2)\dots f(a_n)f(a_{n+1})}}-\frac{n^2}{\sqrt[n]{f(a_1)f(a_2)\dots f(a_n)}}\right)=\frac{e}{ac}$$

9) D.M. Bătineţu - Giurgiu, Neculai Stanciu, SSM 5/2016.

If $a,b\in\mathbb{R}, a+b=1$, then $\lim\limits_{n\to\infty}\left((n+1)^a\sqrt[n+1]{((n+1)!c_n)^b}-n^a\sqrt[n]{(n!e_n)^b}\right)=\frac{a+b\ln c}{e^b}$

where $e_n=\left(1+\frac{1}{n}\right)^n$ and $c_n=-\ln n+\sum\limits_{k=1}^{n}\frac{1}{k}$

10) D.M. Bătineţu - Giurgiu, Neculai Stanciu, P 2/2012.

If $x_n(t)=n^{1-t}\left(\frac{(\sqrt[n+1]{(n+1)!})^{2t}}{(n+1)^t}-\frac{(\sqrt[n]{n!})^{2t}}{n^t}\right)$, with $t>0$, then $\lim\limits_{n\to\infty}x_n(t)=te^{-2t}$.

11) D.M. Bătineţu - Giurgiu, Neculai Stanciu, GMB 5/2012.

If $(I_n(t))_{n\geq 2}, I_n(t)=n^{1-t}\left((n+1)^t(\sqrt[n+1]{n+1})^t-n^t(\sqrt[n]{n})^t\right);\forall t\in\mathbb{R}^*$, then

$$\lim_{n\to\infty}I_n(t)=t$$

12) D.M. Bătineţu - Giurgiu, Neculai Stanciu, CM 7/2013.
If $(a_n)_{n\geq 1}$ is a positive real sequence such that

$\lim\limits_{n\to\infty}\frac{a_{n+1}-a_n}{n}=a\in\mathbb{R}_+^*$ and $a_n!$ is defined by $a_{n+1}=a_n!\cdot a_{n+1},\forall n\in\mathbb{N}^*$, then

$$\lim_{n\to\infty}\left(\frac{\sqrt[n+1]{a_{n+1}!}}{n+1}-\frac{\sqrt[n]{a_n!}}{n}\right)=\frac{a}{2e^2}$$

13) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RM 2/2012.

If $u_n=\frac{(n+2)^{n+1}}{(n+1)^n},\forall n\in\mathbb{N}^*$, then $\lim\limits_{n\to\infty}(\sqrt[n+1]{u_1u_2\dots u_nu_{n+1}}-\sqrt[n]{u_1u_2\dots u_n})=1$

14) D.M. Bătineţu - Giurgiu, Neculai Stanciu, P 2/2011.

If $x_n=\sqrt[n]{\sqrt{2!}\cdot\sqrt[3]{3!}\cdot\dots\cdot\sqrt[n]{n!}}$, then $\lim\limits_{n\to\infty}\left(\frac{(n+1)^2}{x_{n+1}}-\frac{n^2}{x_n}\right)=e^2$.

15) D.M. Bătineţu - Giurgiu, Neculai Stanciu, MP 3/2013.

If $f,g:\mathbb{R}_+^*\to\mathbb{R}_+^*$ such that $\lim\limits_{x\to\infty}(f(x+1)-f(x))=a\in\mathbb{R}_+^*$, $\lim\limits_{x\to\infty}\frac{g(x+1)}{xg(x)}=b\in\mathbb{R}_+^*$

and the following limits exists $\lim\limits_{x\to\infty}\frac{f(x)}{x}$ and $\frac{(g(x))^{\frac{1}{x}}}{x}, t\in\mathbb{R}$, then

$$\lim_{x\to\infty}(f(x))^{\cos^2 t}\left((g(x))^{\frac{\sin^2 t}{x+1}}-(g(x))^{\frac{\sin^2 t}{x}}\right)\frac{a^{\cos^2 t\cdot b^{\sin^2 t}}}{e^{\sin^2 t}}\cdot\sin^2 t$$

16) D.M. Bătineţu - Giurgiu, Neculai Stanciu, MP 3/2013.

If $f, g : \mathbb{R}_+^* \to \mathbb{R}_+^*$ such that $\lim_{x\to\infty}(f(x+1)-f(x)) = a \in \mathbb{R}_+^*$, $\lim_{x\to\infty} \frac{g(x+1)}{xg(x)} = b \in \mathbb{R}_+^*$

and the following limits exists $\lim_{x\to\infty} \frac{f(x)}{x}$, $\lim_{x\to\infty} \frac{(g(x))^{\frac{1}{x}}}{x}$, $t \in \mathbb{R}$, then

$$\lim_{x\to\infty}(f(x))^{\sin^2 t}\left((g(x))^{\frac{\cos^2 t}{x+1}} - (g(x))^{\frac{\cos^2 t}{x}}\right)\frac{a^{\sin^2 t} \cdot b^{\cos^2 t}}{e^{\cos^2 t}} \cdot \cos^2 t$$

17) D.M. Bătineţu - Giurgiu, Neculai Stanciu, AMM 9/2012.

$$\lim_{x\to\infty}\left(x^{\sin^2 t}\left(((\Gamma(x+2))^{\frac{\cos^2 t}{x+1}} - ((\Gamma(x+1))^{\frac{\cos^2 t}{x}}\right)\right) = e^{\cos^2 t} \cdot \cos^2 t, t \in \mathbb{R}$$

and Γ gamma function.

18) D.M. Bătineţu - Giurgiu, Neculai Stanciu, AMM 2/2016.

If $\{a_n\}_{n\geq 0}, a_n = \frac{(n+2)^{n+1}}{(n+1)^n}, x \in \mathbb{R}, \{b_n(x)\}_{n\geq 1}, b_n(x) = n^{\sin^2 x}\left(a_{n+1}^{\cos^2 x} - a_n^{\cos^2 x}\right)$, then

$$\lim_{n\to\infty} b_n(x) = \cos^2 x \cdot e^{\cos^2 x}$$

19) D.M. Bătineţu - Giurgiu, Neculai Stanciu, SSM 5/2013.

If $(a_n)_{n\geq 1}, (b_n)_{n\geq 1}, a_n, b_n \in \mathbb{R}_+^*, \forall n \in \mathbb{N}^*$ such that $\lim_{n\to\infty}(a_{n+1} - a_n) = a \in \mathbb{R}_+^*$ and

$$\lim_{n\to\infty} \frac{b_{n+1}}{nb_n} = b \in \mathbb{R}_+^*, x \in \mathbb{R}, \text{ then}$$

$$\lim_{n\to\infty}\left(a_n^{\sin^2 x}\left(\left(\sqrt[n+1]{b_{n+1}}\right)^{\cos^2 x} - \left(\sqrt[n]{b_n}\right)^{\cos^2 x}\right)\right) = \frac{a^{\sin^2 x} b^{\cos^2 x}}{e^{\cos^2 x}} \cdot \cos^2 x$$

20) D.M. Bătineţu - Giurgiu, Neculai Stanciu, Laurenţiu Duican Contest, 2016.

If $x \in \mathbb{R}, (L_n(x))_{n\geq 2}, L_n(x) = n^{\sin^2 x}\left(\left(\sqrt[n+1]{(n+1)!}\right)^{\cos^2 x} - \left(\sqrt[n]{n!}\right)^{\cos^2 x}\right)$, then

$$\lim_{n\to\infty} L_n(x) = \frac{\cos^2 x}{e^{\cos^2 x}}$$

21) D.M. Bătineţu - Giurgiu, Neculai Stanciu, MP 2/2013.

If $x \in \mathbb{R}, (L_n(x))_{n\geq 2}, L_n(x) = n^{\cos^2 x}\left(\left(\sqrt[n+1]{(n+1)!}\right)^{\sin^2 x} - \left(\sqrt[n]{n!}\right)^{\sin^2 x}\right)$, then

$$\lim_{n\to\infty} L_n(x) = \frac{\sin^2 x}{e^{\sin^2 x}}$$

22) D.M. Bătineţu - Giurgiu, Neculai Stanciu, MP 3/2015.

If $(a_n)_{n\geq 1}, (b_n)_{n\geq 1}, a_n \neq a_{n+1}, b_n \neq b_{n+1}, \forall n \in \mathbb{N}^*$ with $\lim_{n\to\infty} = a \in \mathbb{R}$,

$\lim_{n\to\infty} b_n = b \in \mathbb{R}$; $\lim_{n\to\infty}(n(a_{n+1} - a_n)) = c \in \mathbb{R}$, $\lim_{n\to\infty}(n(b_{n+1} - b_n)) = d \in \mathbb{R}$, and $f, g : \mathbb{R} \to \mathbb{R}$ are derivable functions with continuos derivatives on $\mathbb{R}$, then

$$\lim_{n\to\infty}(n \cdot (f(a_{n+1})g(b_{n+1}) - f(a_n)g(a_n))) = cf'(a)g(b) + df(a)g'(b)$$

23) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RMM 2021.

If $(x_n)_{n\geq 1}, x_n \in \mathbb{R}_+^*, \forall n \in \mathbb{N}^*$ such that $\lim_{n\to\infty}(x_{n+1} - x_n) = x \in \mathbb{R}_+^*$, then

$$\lim_{n\to\infty}\left(x_{n+1}\sqrt[n+1]{n+1} - x_n\sqrt[n]{n}\right) = x$$

24) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RMM 1/2018.

If $(x_n)_{n\geq 1}$ is a positive real sequence $\lim\limits_{n\to\infty}(x_{n+1}-x_n)=x>0$ then

$$\lim_{n\to\infty}\left(\frac{(n+1)x_{n+1}}{\sqrt[n+1]{(2n+1)!!}}-\frac{nx_n}{\sqrt[n]{(2n-1)!!}}\right)=\frac{xe}{2}.$$

25) D.M. Bătineţu - Giurgiu, Neculai Stanciu, MP 3/2015.

$$\lim_{n\to\infty}\left((\sqrt[n]{n!})^{F_{m+1}}(\sqrt[n]{(2n-1)!!})^{F_m}\left(\tan\frac{\pi\sqrt[n+1]{(n+1)!}}{4\sqrt[n]{n!}}-1\right)^{F_{m+2}}\right)=\frac{1}{2^{F_{m+1}}}\left(\frac{\pi}{e}\right)^{F_{m+2}}.$$

26) D.M. Bătineţu - Giurgiu, Neculai Stanciu, SSM 5/2014.

If $a\in\mathbb{R}_+^*, \{E_n\}_{n\geq 0}, E_n=\sum\limits_{k=0}^{n}\frac{1}{k!}$ then $\lim\limits_{n\to\infty}\sqrt[n]{n!}(a^{\sqrt[n]{E_n}-1}-1)=\frac{\ln a}{e}$.

27) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RMM 2021.

If $\{\gamma_n\}_{n\geq 1}, \gamma_n=-\ln n+\sum\limits_{k=1}^{n}\frac{1}{k}, \lim\limits_{n\to\infty}\gamma_n=\gamma$ (i.e. γ Euler - Mascheroni constant), then

$$\lim_{n\to\infty}(\sin\gamma_n-\sin\gamma)\sqrt[n]{n!}=\frac{\cos\gamma}{2e}.$$

28) D.M. Bătineţu - Giurgiu, Neculai Stanciu, REOIM 2013.

If $s,t\in\mathbb{R}$ and $\{L_n(s,t)\}_{n\geq 2}$ is defined by

$L_n(s,t)=(n+1)^s\cdot\sqrt[n+1]{((n+1)!)^t}-n^s\cdot\sqrt[n]{(n!)^t}$ then

$$\lim_{n\to\infty}L_n(s,t)=L(s,t)\begin{cases}0, \text{ if } s+t<1\\ e^{-t}, \text{ if } s+t=1\\ \infty, \text{ if } s+t>1\end{cases}$$

29) D.M. Bătineţu - Giurgiu, Neculai Stanciu, SM 1/2015.

If $E_n=\sum\limits_{k=0}^{n}\frac{1}{k!}$, then $\lim\limits_{n\to\infty}(e-E_n)\cdot(n+1)!=1$.

30) D.M. Bătineţu - Giurgiu, Neculai Stanciu, GMB 2/2014 and P 2/2014.

$$\lim_{n\to\infty}\sqrt{n}\cdot(\sqrt[2(n+1)]{(n+1)!}-\sqrt[2n]{n!})=\frac{1}{2\sqrt{e}}$$

31) D.M. Bătineţu - Giurgiu, Neculai Stanciu, AMM 7/2016.
If $f:\mathbb{R}_+^*\to\mathbb{R}_+^*$ is a continue function such that

$\lim\limits_{x\to\infty}\frac{f(x)}{x^m}=a\in\mathbb{R}_+^*$, where $m\in[1,\infty)$, then

$$\lim_{n\to\infty}\left(\sqrt[n+1]{\prod_{k=1}^{n+1}f(k)}-\sqrt[n]{\prod_{k=1}^{n}f(k)}\right)=\begin{cases}\frac{a}{e}, \text{ for } m=1\\ \infty, \text{ for } m\in(1,\infty)\end{cases}$$

32) D.M. Bătineţu - Giurgiu, Anastasios Kotronis, Neculai Stanciu, SSM 1/2020.

If $f:\mathbb{R}_+^*\to\mathbb{R}_+^*$ is a continue function such that $\lim\limits_{x\to\infty}\frac{f(x)}{x^2}=a\in\mathbb{R}_+^*$, then

$$\lim_{n\to\infty}\left(\sqrt[n+1]{\prod_{k=1}^{n+1}\frac{f(k)}{k}}-\sqrt[n]{\prod_{k=1}^{n}\frac{f(k)}{k}}\right)=\frac{a}{e}.$$

33) If $(a_n)_{n\geq 1}$ is a positive real sequence such that

$$\lim_{n\to\infty}\frac{a_n}{n!}=a>0,\text{ then }\lim_{n\to\infty}\left(\frac{(n+1)^2}{\sqrt[n+1]{a_{n+1}}}-\frac{n^2}{\sqrt[n]{a_n}}\right)=e.$$

34) D.M. Bătineţu - Giurgiu, Neculai Stanciu, PME 1/2015.

If $(a_n)_{n\geq 1}$ is a positive real sequence such that $\lim\limits_{n\to\infty}\frac{a_n}{n!}=a>0$, then

$$\lim_{n\to\infty}\left(\sqrt[n+1]{a_{n+1}}-\sqrt[n]{a_n}\right)=\frac{1}{e}.$$

35) D.M. Bătineţu - Giurgiu, Neculai Stanciu, CM 7/2014.

If $(a_n)_{n\geq 1}$ is a positive real sequence such that $\lim\limits_{n\to\infty}(a_n-a\cdot n!)=b>0 (a>0)$, then

$$\lim_{n\to\infty}\left(\sqrt[n+1]{a_{n+1}}-\sqrt[n]{a_n}\right)=\frac{1}{e}.$$

36) D.M. Bătineţu - Giurgiu, Neculai Stanciu, LG 1/2017.

If $(a_n)_{n\geq 1}$ is a positve real sequences such that $\lim\limits_{n\to\infty}(a_{n+1}-a_n)=a>0$ and $(x_n)_{n\geq 1}$ is defined by $x_1=1, x_n=\sqrt[n]{\sqrt{3!}\cdot\sqrt[3]{5!}\cdot\ldots\cdot\sqrt[n]{(2n-1)!}}$, then

a) $\lim\limits_{n\to\infty}\left(\frac{a_{n+1}^3}{x_{n+1}}-\frac{a_n^3}{x_n}\right)=\frac{a^3e^4}{4}$; b) $\lim\limits_{n\to\infty}\left(\frac{x_{n+1}}{a_{n+1}}-\frac{x_n}{a_n}\right)=\frac{4}{a\cdot e^4}$

37) D.M. Bătineţu - Giurgiu, Neculai Stanciu, REOIM 2017.
If $(a_n)_{n\geq 1}, (b_n)_{n\geq 1}$ are positive real sequences such that $b_n=a_1\cdot\sqrt{a_2}\cdot\sqrt[3]{a_3}\cdot\ldots\cdot\sqrt[n]{a_n}!$ and

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n\cdot n}=a,\text{ then }\lim_{n\to\infty}\left(\frac{(n+1)^2}{\sqrt[n+1]{b_{n+1}}}-\frac{n^2}{\sqrt[n]{b_n}}\right)=\frac{e^2}{a}.$$

38) D.M. Bătineţu - Giurgiu, Neculai Stanciu, SSM 4/2018.

If $(x_n)_{n\geq 1}, x_1=1, x_n=1\cdot\sqrt{3!!}\cdot\sqrt[3]{5!!}\cdot\ldots\cdot\sqrt[n]{(2n-1)!!}$, then

$$\lim_{n\to\infty}\left(\frac{(n+1)^2}{\sqrt[n+1]{x_{n+1}}}-\frac{n^2}{\sqrt[n]{x_n}}\right)=\frac{e^2}{2}.$$

39) D.M. Bătineţu - Giurgiu, Neculai Stanciu, SSM 7/2018.

If $a\in\left(0,\frac{\pi}{2}\right)$ and $b=\arcsin a$, then

$$\lim_{n\to\infty}\sqrt[n]{n!}\left(\sin\left(\frac{b\cdot\sqrt[n+1]{(2n+1)!!}}{\sqrt[n]{(2n-1)!!}}\right)-a\right)=\frac{\sqrt{1-a^2}}{e}\arcsin a$$

40) D.M. Bătineţu - Giurgiu, Neculai Stanciu, SSM 3/2019.

If $a\in\left(0,\frac{\pi}{2}\right)$, then $\lim\limits_{n\to\infty}\sqrt[n]{(2n-1)!!}\left(\sin\left(\frac{a\cdot\sqrt[n+1]{(n+1)!}}{\sqrt[n]{n!}}\right)-\sin a\right)=\frac{2a\cos a}{e}$.

41) D.M. Bătineţu - Giurgiu, Neculai Stanciu, SM 2/2017.
If $a\in(0,1)$ and $b=\arcsin a$, then

$$\lim_{n\to\infty}\sqrt[n]{n!}\left(\sin\left(\frac{b\cdot\sqrt[n+1]{(n+1)!}}{\sqrt[n]{n!}}\right)-a\right)=\frac{\sqrt{1-a^2}}{e}\arcsin a$$

42) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RM 2/2017.

$$\text{If } \gamma_n = -\ln n + \sum_{k=1}^{n} \frac{1}{k}, \lim_{n\to\infty} \gamma_n = \gamma, \text{ then } \lim_{n\to\infty} (\gamma_n - \gamma) \sqrt[n]{(2n-1)!!} = \frac{1}{e}.$$

43) D.M. Bătineţu - Giurgiu, Neculai Stanciu, SM 1/2018.

$$\text{If } s_n = -2\sqrt{n} + \sum_{k=1}^{n} \frac{1}{\sqrt{k}}, \lim_{n\to\infty} s_n = s = \text{ Ioachimescu's constant, then}$$

$$\lim_{n\to\infty} (s_n - s) \sqrt[2n]{(2n-1)!!} = \frac{1}{\sqrt{2e}}$$

44) D.M. Bătineţu - Giurgiu, Neculai Stanciu, CMJ 2/2018.

$$\text{If } s_n = -2\sqrt{n} + \sum_{k=1}^{n} \frac{1}{\sqrt{k}}, \lim_{n\to\infty} s_n = s = \text{ Ioachimescu's constant, then}$$

$$\lim_{n\to\infty} (s_n - s) \sqrt[2n]{n!} = \frac{1}{2\sqrt{e}}.$$

45) D.M. Bătineţu - Giurgiu, Neculai Stanciu, CMJ 3/2018 and RM 1/2018.

$$\text{If } e_n = \left(1 + \frac{1}{n}\right)^n, \lim_{n\to\infty} e_n = e = \text{ Euler constant and } s_n = -2\sqrt{n} + \sum_{k=1}^{n} \frac{1}{\sqrt{k}},$$

$$\lim_{n\to\infty} s_n = s = \text{ Ioachimescu constant, then } \lim_{n\to\infty} \frac{e - e_n}{(s_n - s)^2} = 2e.$$

46) D.M. Bătineţu - Giurgiu, Neculai Stanciu, SSM 2/2020.

$$\lim_{n\to\infty} \frac{1}{(\sqrt[n]{(2n-1)!!})^2} \sum_{k=1}^{n} \left[(\sqrt[2k]{k!} + \sqrt[2(k+1)]{(k+1)!})^2 \right] = \frac{e}{2}, \text{ where } [x] \text{ is integer part of } x.$$

47) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 3/2019.

$$\lim_{n\to\infty} \left(\lim_{n\to\infty} \left((f(x+1))^{\frac{L_n}{(x+1)F_{n+1}}} - (f(x))^{\frac{L_n}{xL_{n+1}}} \right) x^{\frac{L_{n-1}}{L_{n+1}}} \right) = \left(\frac{a}{e}\right)^{\frac{1}{\alpha}} \frac{1}{\alpha}(1 + \ln a),$$

$$\text{where } f : \mathbb{R}_+^* \to \mathbb{R}_+^* \text{ verify } \lim_{x\to\infty} \frac{f(x+1)}{xf(x)} = a \in \mathbb{R}_+^*.$$

48) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 3/2014.

$$\lim_{n\to\infty} (\sqrt[n+1]{(n+1)!F_{n+1}} - \sqrt[n]{n!F_n}) = \frac{\alpha}{e}.$$

49) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 3/2014.

$$\lim_{n\to\infty} (\sqrt[n+1]{(n+1)!L_{n+1}} - \sqrt[n]{n!L_n}) = \frac{\alpha}{e}.$$

50) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 3/2014.

$$\lim_{n\to\infty} (\sqrt[n+1]{(2n+1)!!F_{n+1}} - \sqrt[n]{(2n-1)!!F_n} = \frac{2\alpha}{e}.$$

51) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 3/2014.

$$\lim_{n\to\infty} (\sqrt[n+1]{(2n+1)!!L_{n+1}} - \sqrt[n]{(2n-1)!!L_n}) = \frac{2\alpha}{e}.$$

52) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 4/2014.

If $e_n = \left(1+\frac{1}{n}\right)^n$, with $\lim\limits_{n\to\infty} e_n = e$, then $\lim\limits_{n\to\infty}(e\cdot \sqrt[n+1]{(n+1)!F_{n+1}} - e_n\cdot \sqrt[n]{n!F_n}) = \frac{3\alpha}{2}$.

53) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 4/2014.

$$\text{If } e_n = \left(1+\frac{1}{n}\right)^n, \text{ with } \lim_{n\to\infty} e_n = e, \text{ then}$$

$$\lim_{n\to\infty}(e_{n+1}\cdot \sqrt[n+1]{(n+1)!F_{n+1}} - e_n\cdot \sqrt[n]{n!F_n}) = \alpha$$

54) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 4/2014.

If $e_n = \left(1+\frac{1}{n}\right)^n$, with $\lim\limits_{n\to\infty} e_n = e$, then $\lim\limits_{n\to\infty}(e\cdot \sqrt[n+1]{(n+1)!L_{n+1}} - e_n\cdot \sqrt[n]{n!L_n}) = \frac{3\alpha}{2}$.

55) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 4/2014.

If $e_n = \left(1+\frac{1}{n}\right)^n$, with $\lim\limits_{n\to\infty} e_n = e$, then $\lim\limits_{n\to\infty}(e_{n+1}\cdot \sqrt[n+1]{(n+1)!L_{n+1}} - e_n\cdot \sqrt[n]{n!L_n}) = \alpha$.

56) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 3/2016.

$$\text{If } e_n = \left(1+\frac{1}{n}\right)^n, \text{ with } \lim_{n\to\infty} e_n = e, \text{ then}$$

$$\lim_{n\to\infty}(e_{n+1}\cdot \sqrt[n+1]{(2n+1)!!F_{n+1}} - e_n\cdot \sqrt[n]{(2n-1)!!F_n}) = 2\alpha$$

57) D.M. Bătineţu - Giurgiu, Neculai Stanciu, AMM 10/2015.

$$\text{If } e_n = \left(1+\frac{1}{n}\right)^n, \text{ with } \lim_{n\to\infty} e_n = e, \text{ then}$$

$$\lim_{n\to\infty}(e_{n+1}\cdot \sqrt[n+1]{(2n+1)!!L_{n+1}} - e_n\cdot \sqrt[n]{(2n-1)!!L_n}) = 2\alpha$$

58) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 3/2016.

$$\text{If } e_n = \left(1+\frac{1}{n}\right)^n, \text{ with } \lim_{n\to\infty} e_n = e, \text{ then}$$

$$\lim_{n\to\infty}(e\cdot \sqrt[n+1]{(2n+1)!!F_{n+1}} - e_n\cdot \sqrt[n]{(2n-1)!!F_n}) = 3\alpha$$

59) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 3/2016.

$$\text{If } e_n = \left(1+\frac{1}{n}\right)^n, \text{ with } \lim_{n\to\infty} e_n = e, \text{ then}$$

$$\lim_{n\to\infty}(e\cdot \sqrt[n+1]{(2n+1)!!L_{n+1}} - e_n\cdot \sqrt[n]{(2n-1)!!L_n}) = 3\alpha.$$

60) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 2/2015.

$$\text{If } a,b>0, c\geq 0, \text{ then } \lim_{n\to\infty}\left(\frac{(n+1)^2}{\sqrt[n+1]{(n+1)!aL_{n+1}^c}} - \frac{n^2}{\sqrt[n]{n!bL_n^c}}\right) = \frac{e}{\alpha^c}\left(1+\ln\frac{b}{a}\right)$$

61) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 2/2015.

$$\text{If } a,b>0, c\geq 0, \text{ then } \lim_{n\to\infty}\left(\frac{(n+1)^2}{\sqrt[n+1]{(n+1)!aF_{n+1}^c}} - \frac{n^2}{\sqrt[n]{n!bF_n^c}}\right) = \frac{e}{\alpha^c}\left(1+\ln\frac{b}{a}\right)$$

62) D.M. Bătineţu - Giurgiu, Neculai Stanciu, Gabriel Tica, FQ 1/2017.

$$\text{If } (a_n)_{n\geq 1} \text{ is a positive sequence such that } \lim_{n\to\infty}\frac{a_{n+1}}{n^2 a_n} = a \in \mathbb{R}_+^*, \text{ then}$$

$$\lim_{n\to\infty}\left(\sqrt[n+1]{\frac{a_{n+1}F_{n+1}}{(n+1)!}}-\sqrt[n]{\frac{a_nF_n}{n!}}\right)=\frac{a\alpha}{e}.$$

63) D.M. Bătineţu - Giurgiu, Neculai Stanciu, Gabriel Tica, FQ 1/2017.

If $(a_n)_{n\geq 1}$ is a positive real sequence such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{n^2a_n}=a\in\mathbb{R}_+^*$, then

$$\lim_{n\to\infty}\left(\sqrt[n+1]{\frac{a_{n+1}L_{n+1}}{(n+1)!}}-\sqrt[n]{\frac{a_nL_n}{n!}}\right)=\frac{a\alpha}{e}.$$

64) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RMM 2021.

If $(a_n)_{n\geq 1}$ is a positive real sequence such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{n^2a_n}=a\in\mathbb{R}_+^*$, then

$$\lim_{n\to\infty}\left(\sqrt[n+1]{\frac{a_{n+1}F_{n+1}}{(2n+1)!!}}-\sqrt[n]{\frac{a_nF_n}{(2n-1)!!}}\right)=\frac{a\alpha}{2e}.$$

65) D.M. Bătineţu - Giurgiu, Neculai Stanciu, MP 2/2016.

If $(a_n)_{n\geq 1}$ is a positive real sequence such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{n^2a_n}=a\in\mathbb{R}_+^*$, then

$$\lim_{n\to\infty}\left(\sqrt[n+1]{\frac{a_{n+1}L_{n+1}}{(2n+1)!!}}-\sqrt[n]{\frac{a_nL_n}{(2n-1)!!}}\right)=\frac{a\alpha}{2e}.$$

66) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 1/2018.
If $a,b,c\in\mathbb{R}_+$, then

$$\lim_{n\to\infty}\frac{\left(\sqrt[n+1]{(2n+1)!!F_{n+1}^b}\right)^{a+1}-\left(\sqrt[n]{(2n-1)!!F_n^b}\right)^{a+1}}{\left(\sqrt[n]{n!L_n^c}\right)^a}=\frac{(a+1)2^{a+1}\alpha^{a(b-c)+b}}{e}.$$

67) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 2/2018.
If $m,p\geq 0$, then

$$\lim_{n\to\infty}\left(\frac{\sqrt[n+1]{((2n+1)!!)^{m+1}L_{n+1}^{p(m+1)}}}{(n+1)^m}-\frac{\sqrt[n]{((2n-1)!!)^{m+1}L_n^{p(m+1)}}}{n^m}\right)=\left(\frac{2\alpha^p}{e}\right)^{m+1}$$

68) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 3/2018.

If $(a_n)_{n\geq 0}, a_n\in\mathbb{R}_+^*$ such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{n\cdot a_n}=a\in\mathbb{R}_+^*$, then

$$\lim_{m\to\infty}\left(\lim_{n\to\infty}\left(\left(\left(\sqrt[n+1]{a_{n+1}}\right)^{\frac{F_m}{F_{m+1}}}-\left(\sqrt[n]{a_n}\right)^{\frac{F_m}{F_{m+1}}}\right)n^{\frac{F_{m-1}}{F_{m+1}}}\right)\right)=\left(\frac{a}{e}\right)^{\frac{1}{\alpha}}\cdot\frac{1}{\alpha}$$

69) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 3/2018.

If $(a_n)_{n\geq 0}, a_n\in\mathbb{R}_+^*$ such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{n\cdot a_n}=a\in\mathbb{R}_+^*$, then

$$\lim_{m\to\infty}\left(\lim_{n\to\infty}\left(\left(\left(\sqrt[n+1]{a_{n+1}}\right)^{\frac{L_m}{L_{m+1}}}-\left(\sqrt[n]{a_n}\right)^{\frac{L_m}{L_{m+1}}}\right)n^{\frac{L_{m-1}}{L_{m+1}}}\right)\right)=\left(\frac{a}{e}\right)^{\frac{1}{\alpha}}\cdot\frac{1}{\alpha}$$

70) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 2021.

$$\lim_{m\to\infty}\left(\sqrt[3n+3]{(n+1)!L_{n+1}}-\sqrt[3n]{n!L_n}\right)\sqrt[3]{n^2}=\frac{1}{3}\sqrt[3]{\frac{\alpha}{e}}$$

71) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 2021.

$$\lim_{m\to\infty}\left(\sqrt[3n+3]{(n+1)!F_{n+1}}-\sqrt[3n]{n!F_n}\right)\sqrt[3]{n^2}=\frac{1}{3}\sqrt[3]{\frac{\alpha}{e}}.$$

72) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 1/2020.

$$\lim_{m\to\infty}(\sqrt[3n+3]{(2n+1)!!F_{n+1}}-\sqrt[3n]{(2n-1)!!F_n})\sqrt[3]{n^2}=\frac{1}{3}\sqrt[3]{\frac{2\alpha}{e}}.$$

73) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 2021.

If $(a_n)_{n\geq 1}$ such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{na_n}=a\in\mathbb{R}_+^*, m\in\mathbb{N}^*$, then

$$\lim_{n\to\infty}\left(\sqrt[m(n+1)]{a_{n+1}L_{n+1}}-\sqrt[mn]{a_nL_n}\right)n^{\frac{m-1}{m}}=\frac{1}{m}\left(\frac{a\alpha}{e}\right)^{\frac{1}{m}}.$$

74) D.M. Bătineţu - Giurgiu, Neculai Stanciu, FQ 1/2020.
If $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$ are a positive real sequences such that

$$\lim_{n\to\infty}\frac{a_{n+1}}{n^ra_n}=a\in\mathbb{R}_+^* \text{ and } \lim_{n\to\infty}\frac{b_{n+1}}{n^{s+1}b_n}=b\in\mathbb{R}_+^*, \text{ where } r,s\in\mathbb{R}_+, \text{ then}$$

$$\lim_{n\to\infty}\left(\frac{\sqrt[n]{a_n}\cdot\sqrt[n+1]{F_{n+1}}}{n^{r+s}}-\frac{\sqrt[n+1]{a_{n+1}}\cdot\sqrt[n]{F_n}}{(n+1)^{r+s}}\right)\sqrt[n]{b_n}=\frac{ab\alpha s}{e^{r+s+1}}.$$

75) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RMM 2016.
If $(a_n)_{n\geq 1}$ is a positive real sequence such that

$$\lim_{n\to\infty}(a_{n+1}-a_n)=r\in\mathbb{R}_+^*, u,v\in\mathbb{R} \text{ with } u+v=1,$$

$$a_n!=a_1a_2\dots a_n, G_n=(a_n!)^{\frac{1}{n}}, \forall n\in\mathbb{N}^*, \text{ then}$$

$$\lim_{n\to\infty}\left((n+1)^u\sqrt[n+1]{(G_{n+1}!)^v}-n^u\sqrt[n]{(G_n!)^v}\right)=\left(\frac{r}{e^2}\right)^v$$

76) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RMM 2016.
If $(a_n)_{n\geq 1},(b_n)_{n\geq 1}$ are positive real sequence such that

$$\lim_{n\to\infty}(a_{n+1}-a_n)=a\in\mathbb{R}_+^*, \lim_{n\to\infty}(b_{n+1}-b_n)=b\in\mathbb{R}_+^*, u,v\in\mathbb{R} \text{ with } u+v=1, \text{ then}$$

$$\lim_{n\to\infty}\left(a_{n+1}^u\sqrt[n+1]{(b_1b_2\dots b_nb_{n+1})^v}-a_n^u\sqrt[n]{(b_1b_2\dots b_n)^v}\right)=\frac{a^ub^v}{e^v}.$$

77) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RMM 2016.
If $(a_n)_{n\geq 1}$ is a positive real sequence such that

$$\lim_{n\to\infty}(a_{n+1}-a_n)=r\in\mathbb{R}_+^*, \text{ and } a_n!=a_1a_2\dots a_n, G_n=(a_n!)^{\frac{1}{n}}, \forall n\in\mathbb{N}^*, \text{ then}$$

$$\lim_{n\to\infty}\left(\frac{(n+1)^2}{\sqrt[n+1]{G_{n+1}!}}-\frac{n^2}{\sqrt[n]{G_n!}}\right)=\frac{e^2}{r}.$$

78) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RMM 2016.
If $(a_n)_{n\geq 1},(b_n)_{n\geq 1}$ are positive real sequences such that

$$\lim_{n\to\infty}(a_{n+1}-a_n)=a\in\mathbb{R}_+^*, \lim_{n\to\infty}(b_{n+1}-b_n)=b\in\mathbb{R}_+^*, P_n=\sqrt{\frac{a_1^2+a_2^2+\dots+a_n^2}{n}}$$

and $P_n!=P_1P_2\dots P_n, \forall n\in\mathbb{N}^*, u,v\in\mathbb{R}$, with $u+v=1$, then

$$\lim_{n\to\infty}\left(b_{n+1}^u\sqrt[n+1]{(P_{n+1}!)^v}-b_n^u\sqrt[n]{(P_n!)^v}\right)=\frac{a^vb^u}{e^v(\sqrt{3})^v}.$$

79) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RMM 2016.
If $(a_n)_{n\geq 1}$ is a positive real sequence such that

$$\lim_{n\to\infty}(a_{n+1}-a_n)=r\in\mathbb{R}_+^* \text{ and for any } x\in\mathbb{R}_+^* \text{ denote}$$

$$M_n^{[x]}=\Big(\frac{a_1^x+a_2^x+\ldots+a_n^x}{n}\Big)^{\frac{1}{x}} \text{ and } M_n^{[x]}!=M_1^{[x]}M_2^{[x]}\ldots M_n^{[x]},\forall n\in\mathbb{N}^*, \text{ then}$$

$$\lim_{n\to\infty}\left(\frac{(n+1)^2}{\sqrt[n+1]{M_{n+1}^{[x]}!}}-\frac{n^2}{\sqrt[n]{M_n^{[x]}!}}\right)=\frac{e}{r}(1+x)^{\frac{1}{x}}.$$

80) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RMM 2017.

If $(a_n)_{n\geq 1}$ is a positive real sequence such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{na_n}=a\in\mathbb{R}_+^*$,

$f\in\mathbb{R}[x], f(x)\in\mathbb{R}_+^*, \forall x\in\mathbb{R}_+^*$ and $u,v\in\mathbb{R}$, with $u+v=1$, then

$$\lim_{n\to\infty}\Big((n+1)^u\sqrt[n+1]{(a_{n+1}f(n+1))^v}-n^u\sqrt[n]{(a_nf(n))^v}\Big)=\Big(\frac{a}{e}\Big)^v.$$

81) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RMM 2017.

If $f:\mathbb{R}_+^*\to\mathbb{R}_+^*$ verify $\lim\limits_{n\to\infty}\frac{f(x)}{x}=a\in\mathbb{R}_+^*$ and $(b_n)_{n\geq 1}$ is an arithmetic progression with $b_1,r\in\mathbb{R}_+^*$ and $u,v\in\mathbb{R}$ such that $u+v=1$, then

$$\lim_{n\to\infty}\Big((n+1)^u\sqrt[n+1]{(f(b_1)f(b_2)\ldots f(b_n)f(b_{n+1}))^v}-n^u\sqrt[n]{(f(b_1)f(b_2)\ldots f(b_n))^v}\Big)=\Big(\frac{ar}{e}\Big)^v$$

82) D.M. Bătineţu - Giurgiu, Neculai Stanciu.
If $(x_n)_{n\geq 0}$ is defined by $x_0=0,x_1=1,x_{n+2}=(2n+5)x_{n+1}-(n^2+4n+3)x_n$, $\forall n\in\mathbb{N}$, then

$$\lim_{n\to\infty}(\sqrt[n+1]{L_{n+1}x_{n+1}}-\sqrt[n]{L_nx_n})=\frac{\alpha}{e}.$$

83) D.M. Bătineţu - Giurgiu, Neculai Stanciu.
If $(x_n)_{n\geq 0}$ is defined by $x_0=0,x_1=1,x_{n+2}=(2n+5)x_{n+1}-(n^2+4n+3)x_n$, $\forall n\in\mathbb{N}$, then

$$\lim_{n\to\infty}(\sqrt[n+1]{F_{n+1}x_{n+1}}-\sqrt[n]{F_nx_n})=\frac{\alpha}{e}.$$

84) D.M. Bătineţu - Giurgiu, Neculai Stanciu.
If $(x_n)_{n\geq 0}$ is defined by $(x_n)_{n\geq 0}x_0=0,x_1=1$,
$x_{n+2}=(2n+5)x_{n+1}-(n^2+4n+3)x_n,\forall n\in\mathbb{N}$, then

$$\lim_{n\to\infty}(\sqrt[n+1]{F_{n+1}L_{n+1}x_{n+1}}-\sqrt[n]{F_nL_nx_n})=\frac{\alpha^2}{e}.$$

85) D.M. Bătineţu - Giurgiu, Neculai Stanciu, RMM 2020.

If $(a_n)_{n\geq 1}$ is defined by $a_n=\sum\limits_{k=1}^n\frac{1}{k}$, then $\lim\limits_{n\to\infty}e^{-2a_n}\sum\limits_{k=1}^n\Big[(\sqrt{k}+\sqrt{k+1})^2\Big]=\frac{2}{e^{2\gamma}}$,

where we denoted by $[x]$ the integer part of x.
Remark. Above: $(F_n)_{n\geq 0},F_0=0,F_1=1,F_{n+2}=F_{n+1}+F_n,\forall n\in\mathbb{N}$ is the sequence of Fibonacci; $(L_n)_{n\geq 0},L_0=2,L_1=1,L_{n+2}=L_{n+1}+L_n,\forall n\in\mathbb{N}$ is the sequence of Lucas; $\alpha=\frac{\sqrt{5}+1}{2}$ is golden ratio.

ABBREVIATIONS

Crux Mathematicorum (CM).
Mathematical Gazette B Series (G.M. - B).
La Gaceta de la RSME (LG).
Math Problems (MP).
Pi Mu Epsilon Journal (PME),
Mathematical Magazine from Timişoara (RMT).
Mathematical Recreations (RM).
Escolar Magazine from Iberoamerican Olympiad of Mathematics (REOIM).
Romanian Mathematical Magazine (RMM).
School Science and Mathematics (SSM).
The Brightness of the Mind (SM).
The American Mathematical Monthly (AMM).
The College Mathematics Journal (CMJ).
The Pentagon (P)
The Fibonacci Quarterly (FQ).

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