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JP.345 If $a, b, c \in \mathbb{C}$; $|a| = |b| = |c| = 3$ then:

$$\sum_{cyc} |a + 3| + 3 \sum_{cyc} |a^2 + 1| + \sum_{cyc} |a^3 + 3| \geq 18$$

Proposed by Daniel Sitaru – Romania

Solution by proposer

$$\begin{aligned} 6 &= 2 \cdot 3 = 2 \cdot |a| = 2 \cdot |-a| = |2 \cdot (-a)| = |-2a| = \\ &= |a^3 + 3 - a(a^2 + 1) - (a + 3)| \leq \\ &\leq |a^3 + 3| + |a| \cdot |a^2 + 1| + |a + 3| = \\ &= |a^3 + 3| + 3|a^2 + 1| + |a + 3| \\ &|a + 3| + 3|a^2 + 1| + |a^3 + 3| \geq 6 \end{aligned}$$

By summing:

$$\sum_{cyc} |a + 3| + 3 \sum_{cyc} |a^2 + 1| + \sum_{cyc} |a^3 + 3| \geq 6 + 6 + 6 = 18$$