Simple Approximation

If $n = \frac{(2+\sqrt{3})^{2m-1}+(2-\sqrt{3})^{2m-1}-1}{3}$ and $\sqrt{r} = \frac{(2+\sqrt{3})^{2m-1}-(2-\sqrt{3})^{2m-1}}{2\sqrt{3}}$ where m is any sufficiently large natural number, then

$$\frac{3n+2}{3n^2(n+1)} \approx \frac{n+1}{\sqrt{r}}\Big(tan^{-1}\Big(\frac{4\sqrt{r}}{n^2+4r-1}\Big)\Big) - ln\Big(\frac{n+1}{n}\Big)$$

Eg: If $m = 3$, then

$$\frac{725}{42166806} \approx \frac{22}{19}tan^{-1}\Big(\frac{19}{5291}\Big) - ln\Big(\frac{242}{241}\Big)$$

which is accurate upto 12 digits,
if $m = 4$, then

$$\frac{10085}{113934693606} \approx \frac{82}{71}tan^{-1}\Big(\frac{71}{275561}\Big) - ln\Big(\frac{3362}{3361}\Big)$$

which is accurate upto 18 digits.

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