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ROMANIAN MATHEMATICAL MAGAZINE
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Find:

$$\Omega=\lim_{n\to\infty}\left(\frac{ln(n+1)-lnn}{n+1}+\frac{ln(n+2)-lnn}{n+2}+\frac{ln(n+3)-lnn}{n+3}+\cdots+\frac{ln(n+n)-lnn}{n+n}\right)$$

Proposed by Adil Abdullayev-Baku-Azerbaijan

Solution by Daniel Sitaru-Romania

$$\Omega=\lim_{n\to\infty}\left(\frac{ln(n+1)-lnn}{n+1}+\frac{ln(n+2)-lnn}{n+2}+\frac{ln(n+3)-lnn}{n+3}+\cdots+\frac{ln(n+n)-lnn}{n+n}\right)$$

$$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{ln(n+k)-lnn}{n+k}=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{ln\left(\frac{n+k}{n}\right)}{n+k}=$$

$$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{ln\left(1+\frac{k}{n}\right)}{n\left(1+\frac{k}{n}\right)}\overset{f:[0,1]\to\mathbb{R}}{\triangleq}\lim_{n\to\infty}\sigma_{\Delta_n}(f,\xi_n)\overset{f(x)=\frac{ln(x+1)}{x+1}}{\triangleq}$$

$$\overset{x_n^k=\frac{k}{n}}{\triangleq}\lim_{n\to\infty}\sum_{k=1}^{n}f\left(\frac{k}{n}\right)\left(\frac{k}{n}-\frac{k-1}{n}\right)\overset{\|\Delta_n\|=\frac{1}{n}\to 0}{\triangleq}\int_0^1\frac{ln(x+1)}{x+1}dx=\frac{1}{2}ln^2 2$$