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GENERALIZATION FOR A MARIAN URSĂRESCU INEQUALITY

InΔABC the following relationship holds:

$$\left(\frac{m_a m_b}{m_a+m_b}\right)^n+\left(\frac{m_b m_c}{m_b+m_c}\right)^n+\left(\frac{m_c m_a}{m_c+m_a}\right)^n\ge 3\cdot\left(\frac{3r}{2}\right)^n, n\in\mathbb{N}$$

Proposed by Marin Chirciu-Romania

Solution by Tran Hong-Dong Thap-Vietnam

$$Let\ x=m_a,\ y=m_b,\ z=m_c\rightarrow\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\le\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}=\frac{1}{r}$$

$$\rightarrow LHS=\sum\left(\frac{xy}{x+y}\right)^n\ge\frac{\left(\sum\frac{xy}{x+y}\right)^n}{3^{n-1}}=\frac{\left(\sum\frac{1}{\frac{1}{x}+\frac{1}{y}}\right)^n}{3^{n-1}}$$

$$\overset{Schwarz}{\ge}\frac{\left(\frac{9}{2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)}\right)^n}{3^{n-1}}\ge\frac{\left(\frac{9}{2}\cdot\frac{1}{\frac{1}{r}}\right)^n}{3^{n-1}}=\frac{\left(\frac{9r}{2}\right)^n}{3^{n-1}}=3.\left(\frac{3r}{2}\right)^n$$