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In $\Delta ABC, \mu(A) = 90^{\circ}$ if and only if:

$$\begin{vmatrix} \sqrt{2R} & -\sqrt{b} & \sqrt{2r} & \sqrt{c} \\ \sqrt{b} & -\sqrt{2R} & \sqrt{c} & \sqrt{2r} \\ \sqrt{2r} & \sqrt{c} & -\sqrt{2R} & \sqrt{b} \\ \sqrt{c} & \sqrt{2r} & -\sqrt{b} & \sqrt{2R} \end{vmatrix} = 0$$

Proposed by Radu Diaconu-Romania

Solution by George Florin Şerban-Romania

$$\Delta = \begin{vmatrix} \sqrt{2R} & -\sqrt{b} & \sqrt{2r} & \sqrt{c} \\ \sqrt{b} & -\sqrt{2R} & \sqrt{c} & \sqrt{2r} \\ \sqrt{2r} & \sqrt{c} & -\sqrt{2R} & \sqrt{b} \\ \sqrt{c} & \sqrt{2r} & -\sqrt{b} & \sqrt{2R} \end{vmatrix} = (-1)^{1+1}\sqrt{2R}\begin{vmatrix} -\sqrt{2R} & \sqrt{c} & \sqrt{2r} \\ \sqrt{c} & -\sqrt{2R} & \sqrt{b} \\ \sqrt{2r} & -\sqrt{b} & \sqrt{2R} \end{vmatrix} +$$

$$+(-1)^{1+2}(-\sqrt{b})\begin{vmatrix} \sqrt{b} & \sqrt{c} & \sqrt{2r} \\ \sqrt{2r} & -\sqrt{2R} & \sqrt{b} \\ \sqrt{c} & -\sqrt{b} & \sqrt{2c} \end{vmatrix} + (-1)^{1+3}\sqrt{2r}\begin{vmatrix} \sqrt{b} & -\sqrt{2R} & \sqrt{2r} \\ \sqrt{2r} & \sqrt{c} & \sqrt{b} \\ \sqrt{c} & \sqrt{2r} & \sqrt{2R} \end{vmatrix} +$$

$$+(-1)^{1+4}\sqrt{c}\begin{vmatrix} \sqrt{b} & -\sqrt{2R} & \sqrt{c} \\ \sqrt{2r} & \sqrt{c} & -\sqrt{2R} \\ \sqrt{c} & \sqrt{2r} & -\sqrt{b} \end{vmatrix} =$$

$$= 4R^2 + 4r^2 + 8Rr + b^2 + c^2 + 2bc - 4bR - 4cR - 4br - 4cr$$

$$a = 2R \Rightarrow \Delta = a^2 + b^2 + c^2 + 4r^2 + 8Rr + 2bc - 4R(b + c) - 4r(b + c)$$

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$$S = \frac{bc}{2} \Rightarrow rs = \frac{bc}{2} \Rightarrow bc = 2rs = 2r(R + r), s = 2R + r, \mu(A) = 90^{\circ}$$

$$\Rightarrow \Delta = 2(s^2 - r^2 - 4Rr) + 4r^2 + 8Rr + 4r(2R + r) - 4R(a + b + c) + 4Ra$$

$$- 4r(a + b + c) + 4ra =$$

$$= 2(2R + r)^2 + 6r^2 + 8Rr - 8R(2R + r) + 8R^2 - 8r(2R + r) + 8Rr = 0$$

Note by editor:

Many thanks to Florică Anastase-Romania for typed solution.