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ROMANIAN MATHEMATICAL MAGAZINE

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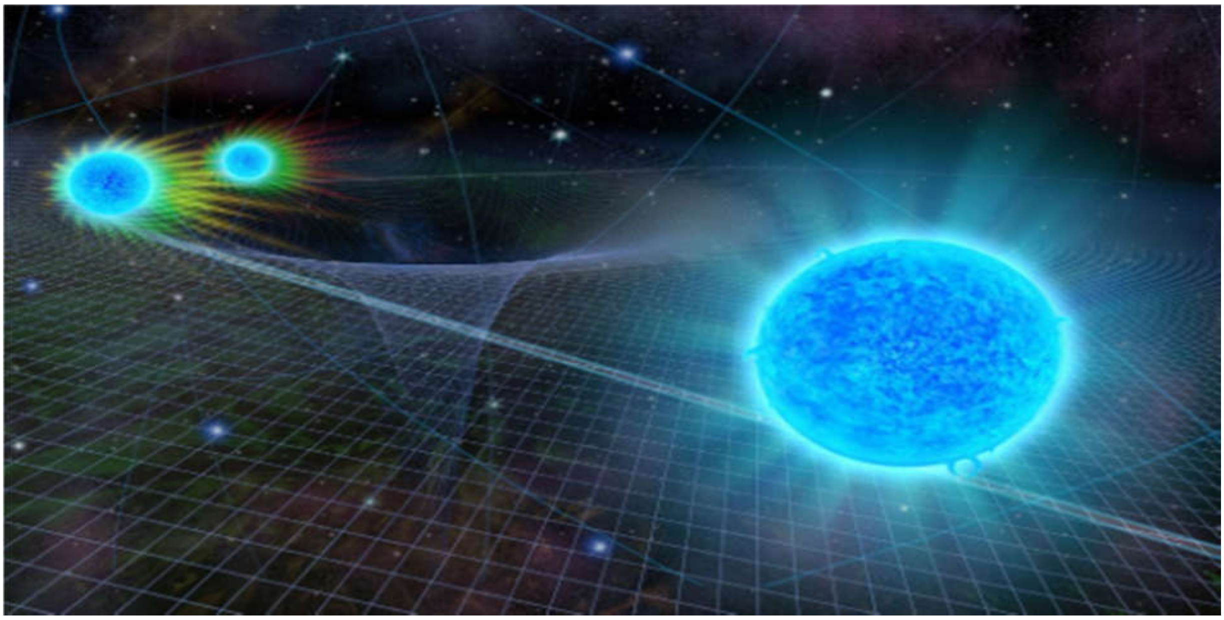

$a, b, c \in \mathbb{C}^*$ −different in pairs, $|a| = |b| = |c|, A(a), B(b), C(c)$. Prove that:

$$\sum_{cyc} \left| \frac{(a+b)(a+c)}{(a-b)(a-c)} \right| = 1 \Leftrightarrow AB = BC = CA$$

Proposed by Marian Ursărescu-Romania

Solution by proposer

$$\sum_{cyc} \left| \frac{(a+b)(a+c)}{(a-b)(a-c)} \right| = 1 \Leftrightarrow \sum_{cyc} \frac{BH \cdot CH}{AB \cdot AC} = 1 \Leftrightarrow \sum_{cyc} \frac{BH \cdot CH}{BC} = 1 \Leftrightarrow$$

$$\sum_{cyc} aBH \cdot CH = abc; (1)$$

But $\sum_{cyc} aBH \cdot CH \geq abc; (2)$ true from

$$aPB \cdot PC + bPC \cdot PA + cPA \cdot PB \geq abc, \forall P \in \mathcal{P}$$

From $(1)\&(2)$ it following that ΔABC is equilateral.

Note by editor:

Many thanks to Florică Anastase-Romania for typed solution.