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$a, b, c \in \mathbb{C}^*$ −different in pairs, $|a| = |b| = |c|, A(a), B(b), C(c)$. Prove that:

$$\sum_{cyc} \frac{|(a-b)(a-c)|}{|(a-b)|a-c| + (a-c)|a-b||^2} = 9\left(\sum_{cyc} |a-b|\right)^{-2} \Leftrightarrow AB = BC = CA$$

Proposed by Marian Ursărescu-Romania

Solution by proposer

$$A(a), B(b), C(c) \Rightarrow \Delta ABC \subset C(O, R), |a| = R$$

$$|a-b| = AB = c', |a-c| = AC = b', |b-c| = BC = a' \Rightarrow$$

$$\sum_{cyc} \frac{AB \cdot AC}{|(a-b)AC + (a-c)AB|^2} = \frac{9}{(AB+BC+CA)^2} \Leftrightarrow$$

$$\sum_{cyc} \frac{bc}{\left|\frac{(a-b)a' + (a-c)c'}{a+b+c}\right|^2} = 9 \Leftrightarrow \sum_{cyc} \frac{bc}{\left|\frac{(b+c)a' - bb' - cc'}{a+b+c}\right|^2} = 9 \Leftrightarrow$$

$$\sum_{cyc} \frac{bc}{\left|\frac{(a+b+c)a' - aa' - bb' - cc'}{a+b+c}\right|^2} = 9 \Leftrightarrow \sum_{cyc} \frac{bc}{\left|a' - \frac{aa' + bb' + cc'}{a+b+c}\right|^2} = 9$$

$$\Rightarrow \sum_{cyc} \frac{bc}{AI^2} = 9; (1)$$

But $\sum_{cyc}\frac{bc}{AI^2}=\frac{4R+r}{r}\geq 9\Leftrightarrow 4R+r\geq 9r\Leftrightarrow R\geq 2r;(2)$

From $(1)\&(2)$ **we have equality, then** ΔABC **equilateral.**

Note by editor:
Many thanks to Florică Anastase-Romania for typed solution.