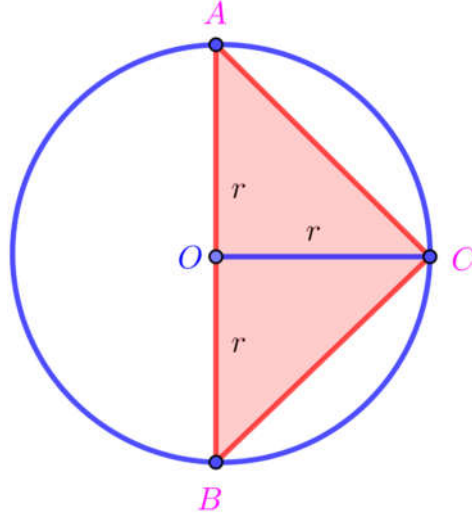


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$a, b, c \in \mathbb{C}^*$ –different in pairs, $|a| = |b| = |c| = 1$, $a + b + c \in \{\pm 1, \pm i\}$.

Find:

$$\Omega = a^{-2021} + b^{-2021} + c^{-2021}$$

Proposed by Marian Ursărescu-Romania

Solution by Ravi Prakash-New Delhi-India

$a + b + c$ represents orthocentre of $\triangle ABC$

$a + b + c \in \{\pm 1, \pm i\} \Rightarrow a + b + c$ lies on the circumcircle of $\triangle ABC \Rightarrow \triangle ABC$ is right triangle.

If $a + b + c = \pm 1$, then one of the vertices is 1 or -1 and other two $i, -i$

$$\Omega = a^{-2021} + b^{-2021} + c^{-2021} = i^{-2021} + (-i)^{-2021} + (\pm i)^{2021} = \pm i$$

If $a + b + c = \pm i$, then one of the vertices is i or $-i$ and other two are 1, -1 .

$$\Omega = a^{-2021} + b^{-2021} + c^{-2021} = 1^{-2021} + (-1)^{2021} + (\pm i)^{-2021} = \pm i$$

Therefore,

$$\Omega = a^{-2021} + b^{-2021} + c^{-2021} \in \{\pm 1, \pm i\}.$$

Note by editor:

Many thanks to Florică Anastase-Romania for typed solution.