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**In $\Delta ABC$ the following relationship holds:**

$$\left(\sum_{cyc}\sqrt{m_a}\right)\left(\sum_{cyc}\sqrt{w_a}\right)\left(\sum_{cyc}m_a\right)\left(\sum_{cyc}w_a\right)\geq\frac{27}{2R+5r}\left(\sum_{cyc}h_ah_b\right)^2$$

***Proposed by Mokhtar Khassani-Mostaganem-Algerie***

***Solution by Marian Ursărescu-Romania***

$$\sum_{cyc}\sqrt{m_a}\geq 3\sqrt[3]{\sqrt{m_am_bm_c}}\text{ and }\sum_{cyc}m_a\geq 3\sqrt[3]{m_am_bm_c}\Rightarrow$$

$$\left(\sum_{cyc}\sqrt{m_a}\right)\left(\sum_{cyc}m_a\right)\geq 9\sqrt{m_am_bm_c}$$

**Similarly:**

$$\left(\sum_{cyc}\sqrt{w_a}\right)\left(\sum_{cyc}w_a\right)\geq 9\sqrt{w_aw_bw_c}$$

**Therefore,**

$$\left(\sum_{cyc}\sqrt{m_a}\right)\left(\sum_{cyc}\sqrt{w_a}\right)\left(\sum_{cyc}m_a\right)\left(\sum_{cyc}w_a\right)>81\sqrt{m_am_bm_cw_aw_bw_c}$$

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**We must show that:**

$$3\sqrt{m_a m_b m_c w_a w_b w_c} \geq \frac{1}{2R+5r} \cdot \left(\sum_{cyc} h_a h_b\right)^2; \ \ (1)$$

**But in any** $\Delta ABC$ **we have:** $m_a \cdot w_a \geq s(s-a); \left(\therefore s = \frac{a+b+c}{2}\right) \Rightarrow$

$$\sqrt{m_a m_b m_c w_a w_b w_c} \geq sS = s^2 r; \ \ (2)$$

**From (1),(2) we must show:**

$$3s^2 r \geq \frac{1}{2R+5r} \cdot \left(\sum_{cyc} h_a h_b\right)^2; \ \ (3)$$

**But** $\sum_{cyc} h_a h_b = \frac{2s^2 r}{R}; \ \ (4)$

**From (3),(4) we must show:** $3s^2 r \geq \frac{1}{2R+5r} \cdot \frac{4s^4 r^2}{R^2} \Leftrightarrow 3R^2(2R+5r) \geq 4s^2 r; \ \ (5)$

**From Mitrinovic inequality:** $s^2 \leq \frac{27R^2}{4}; \ \ (6)$

**From (5),(6) we must show:** $3(2R+5r) \geq 27r \Leftrightarrow 2R+5r \geq 9r \Leftrightarrow 2R \geq 4r \Leftrightarrow$

$R \geq 2r \ (Euler)$ **true.**

**Proved.**

**Note by editor:**

**Many thanks to Florică Anastase-Romania for typed solution.**