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Solve for natural numbers:

$$(2n - 3) \cdot \prod_{k=3}^n \frac{8k^3 - 12k^2 - 26k + 15}{8k^3 + 12k^2 - 26k - 15} = 10n - 65$$

Proposed by Costel Florea-Romania

Solution 1 by Rovsen Pirguliyev-Sumgait-Azerbaijan, Solution 2 by Izumi Ainsworth-Lima-Peru

Solution 1 by Rovsen Pirguliyev-Sumgait-Azerbaijan

Since

$$8k^3 - 12k^2 - 26k + 15 = (2k - 5)(2k - 1)(2k + 3)$$

$$8k^3 + 12k^2 - 26k - 15 = (2k - 3)(2k + 1)(2k + 8)$$

We have:

$$(2n - 3) \cdot \prod_{k=3}^n \frac{8k^3 - 12k^2 - 26k + 15}{8k^3 + 12k^2 - 26k - 15} = 10n - 65$$

$$(2n - 3) \cdot \frac{1 \cdot 5 \cdot 9}{3 \cdot 7 \cdot 11} \cdot \frac{3 \cdot 7 \cdot 11}{5 \cdot 9 \cdot 14} \cdots \frac{(2n - 7)(2n - 3)(2n + 1)}{(2n - 5)(2n - 1)(2n + 3)}$$

$$\cdot \frac{(2n - 5)(2n - 1)(2n + 3)}{(2n - 3)(2n + 1)(2n + 5)} = 10n - 65$$

Hence,

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$$(2n - 3) \cdot \frac{1}{(2n + 5)} = 10n - 65$$

$$(2n + 5)(10n - 65) = 2n - 3$$

$$10n^2 - 41n - 161 = 0$$

No solution for natural numbers.

Solution 2 by Izumi Ainsworth-Lima-Peru

$$(2n - 3) \cdot \prod_{k=3}^n \frac{8k^3 - 12k^2 - 26k + 15}{8k^3 + 12k^2 - 26k - 15} = 10n - 65$$

$$(2n - 3) \cdot \prod_{k=3}^n \frac{(2k - 5)(2k - 1)(2k + 3)}{(2k - 3)(2k + 1)(2k + 8)} = 10n - 65$$

$$(2n - 3) \prod_{k=3}^n \frac{2k - 5}{2k + 5} \prod_{k=3}^n \frac{2k - 1}{2k + 1} \prod_{k=3}^n \frac{2k + 3}{2k - 3} = 10n - 65$$

$$(2n - 3) \cdot \frac{9!! (2n - 5)!!}{(2n + 5)!!} \cdot \frac{5}{2n + 1} \cdot \frac{(2n + 3)!!}{7!! (2n - 3)!!} = 10n - 65$$

$$\frac{9}{(2n + 1)(2n + 5)} = 2n - 13$$

No solution for natural numbers.

Note by editor:

Many thanks to Florică Anastase-Romania for typed solutions.