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In ΔABC then following relationship holds:

$$\frac{1}{h_a^2h_b}+\frac{1}{h_b^2h_c}+\frac{1}{h_c^2h_a}\geq\frac{2}{3R}\left(\frac{1}{h_a^2}+\frac{1}{h_b^2}+\frac{1}{h_c^2}\right)$$

Proposed by George Florin Şerban-Romania

Solution by Tran Hong-Dong Thap-Vietnam

$$\frac{1}{h_a^2h_b}+\frac{1}{h_b^2h_c}+\frac{1}{h_c^2h_a}\overset{AGM}{\geq}3\sqrt[3]{\frac{1}{(h_ah_bh_b)^3}}=\frac{3}{h_ah_bh_b}=\frac{3}{\frac{2S^2}{R}}=\frac{3R}{2S^2}$$

$$\frac{2}{3R}\left(\frac{1}{h_a^2}+\frac{1}{h_b^2}+\frac{1}{h_c^2}\right)=\frac{2}{3R}\cdot\frac{a^2+b^2+c^2}{4S^2}$$

We must show that:

$$\frac{3R}{2S^2}\geq\frac{2}{3R}\cdot\frac{a^2+b^2+c^2}{4S^2}\Leftrightarrow 9R^2\geq a^2+b^2+c^2$$

which is true by Leibniz' Inequality.Proved.

Note by editor:

Many thanks to Florică Anastase-Romania for typed solution.