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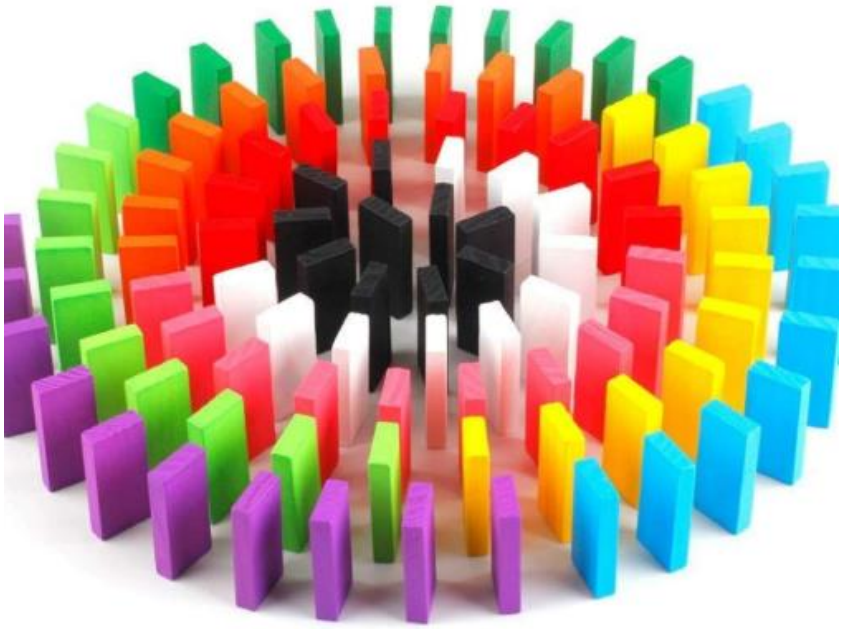

$S(n)$ =sum of digits of positive integer n.Find:

$$\Omega = S\left(S\left(S(2020^{2020})\right)\right)$$

Proposed by Rajeev Rastogi-India

Solution by Florentin Vişescu-Romania

It is obvious: $S(2020^{2020}) = S(202^{2020})$

But: $202 < 1000 = 10^3 \rightarrow 202^{2020} < 10^{6060} \rightarrow$

$$S(202^{2020}) \leq 6060 \cdot 9 = 54540$$

$$S\left(S(2020^{2020})\right) \leq S(49999) = 40 \rightarrow S\left(S(202^{2020})\right) \leq S(39) = 12$$

$$S\left(S\left(S(202^{2020})\right)\right) \leq 12 \rightarrow S\left(S(2020^{2020})\right) \leq 12; (1)$$

But: $S\left(S(2020^{2020})\right)$ give the same rest to the division by 9 like her $2020^{2020} \rightarrow$

$$2020^{2020} \equiv 4(mod9) \rightarrow S\left(S\left(S(202^{2020})\right)\right) \equiv 4(mod9) \rightarrow$$

$$S\left(S\left(S(202^{2020})\right)\right) = 4$$

Note by editor:

Many thanks to Florică Anastase-Romania for typed solutions.