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Solve for natural numbers:

$$\frac{(x-2)!!(x-3)!!}{(x-4)!!(x-5)!!} + \frac{(x-3)!!(x-4)!!}{(x-5)!!(x-6)!!} + \frac{(x-4)!!(x-5)!!}{(x-6)!!(x-7)!!} = 38$$

Proposed by Daniel Sitaru-Romania

Solution 1 by Bedri Hajrizi-Mitrovica-Kosovo, Solution 2 by Rovsen Pirguliev-Sumgait-Azerbaijan

Solution 1 by Bedri Hajrizi-Mitrovica-Kosovo

$$\frac{(x-2)!!(x-3)!!}{(x-4)!!(x-5)!!} + \frac{(x-3)!!(x-4)!!}{(x-5)!!(x-6)!!} + \frac{(x-4)!!(x-5)!!}{(x-6)!!(x-7)!!} = 38 \Leftrightarrow$$

$$(x-2)(x-3) + (x-3)(x-4) + (x-4)(x-5) = 38 \Leftrightarrow$$

$$3x^2 - 21x = 0 \Leftrightarrow 3x(x-7) = 0, x \in \mathbb{N} \Leftrightarrow x = 7.$$

Solution 2 by Rovsen Pirguliev-Sumgait-Azerbaijan

$$\text{Since } (x-2)!! = \underbrace{\dots (x-4)}_{\text{product of even numbers}} \cdot (x-2)$$

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$$(x-3)!! = \dots \underbrace{(x-5)} \cdot (x-3)$$

$$\text{Similarly: } (x-4)!! = \dots \underbrace{(x-6)} \cdot (x-4)$$

$$(x-5)!! = \dots \underbrace{(x-7)} \cdot (x-5) \text{ we have}$$

$$(x-2)(x-3) + (x-3)(x-4) + (x-4)(x-5) = 38 \Leftrightarrow$$

$$3x^2 - 21x = 0 \Leftrightarrow 3x(x-7) = 0, x \in \mathbb{N} \Leftrightarrow x = 7.$$

Note by editor:

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