

PROPOSED PROBLEM

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Without the aid of a computer, find the value of

$$\sum_{n=1}^{+\infty} \frac{15}{25n^2 + 45n - 36}$$

Solution 1 by Narendra Bhandari-Bajura National College-Nepal.

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{15}{25n^2 + 45n - 36} &= \sum_{n=1}^{\infty} \frac{15}{(5n-3)(5n+12)} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{5n-3} - \frac{1}{5n+12} \right) \\ &= \frac{1}{2} + \frac{1}{7} + \frac{1}{12} + \sum_{n=1}^{\infty} \left(\frac{1}{5n+12} - \frac{1}{5n+12} \right) = \frac{61}{84} \end{aligned}$$

The latter sum we have is Telescoping series. □

Solution 2 by Narendra Bhandari-Bajura National College-Nepal.

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{15}{25n^2 + 45n - 36} &= \sum_{n=1}^{\infty} \frac{15}{(5n-3)(5n+12)} \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{5n-3} - \frac{1}{5n+12} \right) \\ &= \frac{1}{2} + \frac{1}{7} + \frac{1}{12} + \sum_{n=1}^{\infty} \left(\frac{1}{5n+12} - \frac{1}{5n+12} \right) = \frac{61}{84} \end{aligned}$$

The latter sum we have is Telescoping series.

After partial fraction decomposition we arrive at

$$\begin{aligned} \sum_{n=1}^{\infty} \left(\frac{1}{5n-3} - \frac{1}{5n+12} \right) &= \sum_{n=0}^{\infty} \left(\frac{1}{5n+2} - \frac{1}{5n+17} \right) \\ &= \sum_{n=0}^{\infty} \left(\frac{1}{5n+5} - \frac{1}{5n+17} \right) - \sum_{n=0}^{\infty} \left(\frac{1}{5n+5} - \frac{1}{5n+2} \right) \\ &= \frac{1}{5} \left(\varphi\left(\frac{17}{5}\right) - \varphi\left(\frac{2}{5}\right) \right) \end{aligned}$$

□

Solution 3 by Alexis Llanos-Catolica Colegio-Lima-Peru.

$$\begin{aligned}
 \sum_{n=1}^{\infty} \frac{15}{\frac{25n^2+45-36}{5n} \frac{+12}{-3}} &= \sum_{n=1}^{\infty} \frac{15}{(5n-3)(5n+12)} \\
 &= \sum_{n=1}^{\infty} \frac{1}{5n-3} - \frac{1}{5n+12} \\
 &= \sum_{n=1}^{\infty} \frac{1}{5n-3} - \frac{1}{5n+2} \dagger \frac{1}{5n+2} - \frac{1}{5n+7} \dagger \frac{1}{5n+7} - \frac{1}{5n+12} \\
 &= \underbrace{\sum_{n=1}^{\infty} \left(\frac{1}{5n-3} - \frac{1}{5n+2} \right)}_V \dagger \underbrace{\sum_{n=1}^{\infty} \left(\frac{1}{5n+2} - \frac{1}{5n+7} \right)}_V \dagger \underbrace{\sum_{n=1}^{\infty} \left(\frac{1}{5n+7} - \frac{1}{5n+12} \right)}_V \\
 &= \frac{1}{2} \dagger \frac{1}{7} \dagger \frac{1}{12} \\
 &= \frac{61}{84}
 \end{aligned}$$

□

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