IONESCU-NESBITT INEQUALITY REVISITED

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Let be $a, y \in \mathbb{R}_+, n \in \mathbb{N}, n \geq 2$ and $b, c, d, x \in \mathbb{R}_+^* = (0, \infty), k = \overline{1, n}$, $X_n = \sum_{k=1}^n x_k$ such that $c \cdot X_n > d \cdot \max_{1 \leq k \leq n}\{x_k\}$. We show that inequality:

$$\sum_{k=1}^n \frac{aX_n + bx_k}{cX_n - dx_k + y} \geq \frac{(an+b)nX_n}{(cn-d)X_n + ny} \tag{1}$$

Proof. We have:

$$\sum_{k=1}^n \frac{aX_n + bx_k}{cX_n - dx_k + y} = \sum_{k=1}^n \frac{aX_n + bx_k}{cX_n - dx_k + y} + \frac{nb}{d} - \frac{nb}{d} = \sum_{k=1}^n \Big(\frac{aX_n + bx_k}{cX_n - dx_k + y} + \frac{b}{d}\Big) - \frac{nb}{d} =$$

$$= \frac{1}{d}\sum_{k=1}^n \frac{adX_n + bdx_k + bcX_n - bdx_k + by}{cX_n - dx_k + y} - \frac{nb}{d}$$

$$= \frac{1}{d}((ad+bc)X_n + by) \cdot \sum_{k=1}^n \frac{1}{cX_n - dX_k + y} - \frac{nb}{d} \overset{\text{Bergström}}{\geq}$$

$$\geq \frac{1}{d}((ad+bc)X_n + by) \cdot \frac{n^2}{\sum_{k=1}^n (cX_n - dx_k + y)} - \frac{nb}{d}$$

$$= \frac{1}{d}((ad+bc)X_n + by) \cdot \frac{n^2}{cnX_n - d\sum_{k=1}^n x_k + ny} - \frac{nb}{d}$$

$$= \frac{1}{d}((ad+bc)X_n + by) \cdot \frac{n^2}{cnX_n - dX_n + ny} - \frac{nb}{d}$$

$$= \frac{1}{d}\Big(((ad+bc)X_n + by)n^2 \cdot \frac{1}{(cn-d)X_n + ny} - bn\Big)$$

$$= \frac{1}{d((cn-d)X_n + ny} \cdot (adn^2X_n + bcn^2X_n + bn^2y - bcn^2X_n + bdnX_n - bn^2y)$$

$$= \frac{1}{d((cn-d)X_n + ny)} \cdot (an+b)dnX_n = \frac{(an+b)nX_n}{(cn-d)X_n + ny} \tag{2}$$

□

Namely the inequality from problem PP.32529 by Mihály Bencze in the Octogon Mathematical Magazine, Vol.27, No.2, October, 2019, page 1106.
In (2) we take $y = 0$, we get:

$$\sum_{k=1}^n \frac{x_k}{X_n - x_k} \geq \frac{n}{n-1} \tag{I-N}$$

Ionescu - Nesbitt inequality for n - variable.

References

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