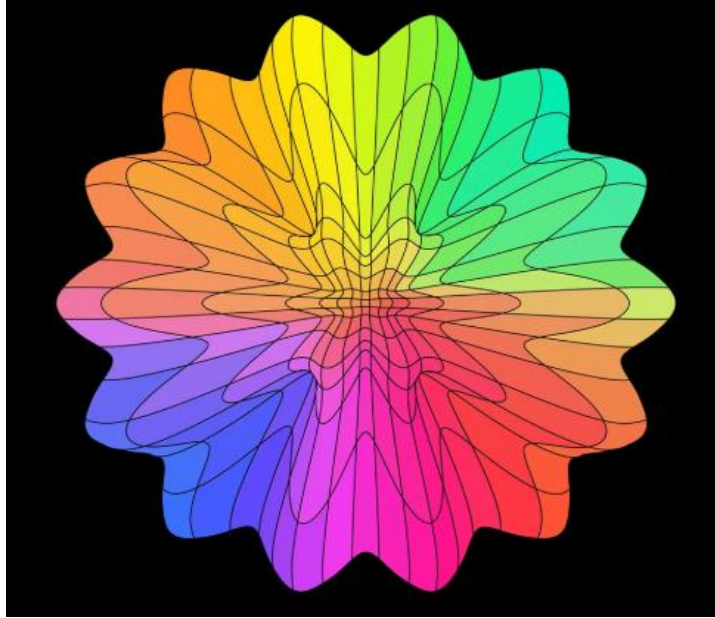


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Find without softs:

$$\Omega = \int_0^{\frac{2}{5}} \frac{\cos^2 x}{\cos^2 \left(x - \frac{1}{5}\right)} dx$$

Proposed by Radu Diaconu-Romania

Solution 1 by Kamel Benaicha-Algiers-Algerie; Solution 2 by Rovsen Pirguliev-Sumgait-Azerbaijan

Solution 1 by Kamel Benaicha-Algiers-Algerie

$$\Omega = \int_0^{\frac{2}{5}} \frac{\cos^2 x}{\cos^2 \left(x - \frac{1}{5}\right)} dx \stackrel{t = \frac{1}{5} - x}{=} \int_{-\frac{1}{5}}^{\frac{1}{5}} \frac{\cos^2 \left(\frac{1}{5} - t\right)}{\cos^2 t} dt = (*)$$

$$\cos^2 \left(\frac{1}{5} - t\right) = \frac{1 + \cos \left(\frac{2}{5} - 2t\right)}{2} = \frac{1 + \cos \left(\frac{2}{5}\right) \cos(2t) + \sin \left(\frac{2}{5}\right) \sin(2t)}{2}$$

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$$\begin{aligned}
 (*) &= \frac{1}{2} \left(\int_{-\frac{1}{5}}^{\frac{1}{5}} \frac{dt}{\cos^2 t} + \cos\left(\frac{2}{5}\right) \int_{-\frac{1}{5}}^{\frac{1}{5}} \frac{2\cos^2 t - 1}{\cos^2 t} dt + 2\sin\left(\frac{2}{5}\right) \int_{-\frac{1}{5}}^{\frac{1}{5}} \tan t dt \right) \\
 &= \tan\left(\frac{1}{5}\right) + \frac{2}{5} \cos\left(\frac{2}{5}\right) - \cos\left(\frac{2}{5}\right) \tan\left(\frac{1}{5}\right) = 2\sin^2\left(\frac{1}{5}\right) \tan\left(\frac{1}{5}\right) + \frac{2}{5} \cos\left(\frac{2}{5}\right) \\
 &= \frac{2}{5} \cos\left(\frac{2}{5}\right) + 2\sin\left(\frac{1}{5}\right) \cos\left(\frac{1}{5}\right) \frac{\sin\left(\frac{1}{5}\right)}{\cos\left(\frac{1}{5}\right)} \tan\left(\frac{1}{5}\right) \\
 &= \frac{2}{5} \cos\left(\frac{2}{5}\right) + \sin\left(\frac{2}{5}\right) \tan^2\left(\frac{1}{5}\right)
 \end{aligned}$$

Solution 2 by Rovsen Pirguliev-Sumgait-Azerbaijan

Denote: $x - \frac{1}{5} = t \Rightarrow x = t + \frac{1}{5}, dx = dt$

$$\begin{aligned}
 \Omega &= \int_0^{\frac{2}{5}} \frac{\cos^2 x}{\cos^2\left(x - \frac{1}{5}\right)} dx = \int_{-\frac{1}{5}}^{\frac{1}{5}} \frac{\cos^2\left(t + \frac{1}{5}\right)}{\cos^2 t} dt \\
 &= \int_{-\frac{1}{5}}^{\frac{1}{5}} \left(\frac{\cos t \cos \frac{1}{5} - \sin t \sin \frac{1}{5}}{\cos t} \right)^2 dt = \int_{-\frac{1}{5}}^{\frac{1}{5}} \left(\cos \frac{1}{5} - \sin \frac{1}{5} \tan t \right)^2 dt \\
 &= \int_{-\frac{1}{5}}^{\frac{1}{5}} \left(\cos^2 \frac{1}{5} - \sin \frac{2}{5} \tan t + \sin^2 \frac{1}{5} \tan^2 t \right) dt \\
 &= \int_{-\frac{1}{5}}^{\frac{1}{5}} \cos^2 \frac{1}{5} dt + \sin \frac{2}{5} \cdot \int_{-\frac{1}{5}}^{\frac{1}{5}} \tan t dt + \sin^2 \frac{1}{5} \cdot \int_{-\frac{1}{5}}^{\frac{1}{5}} \tan^2 t dt
 \end{aligned}$$

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$$\begin{aligned} \int_{-\frac{1}{5}}^{\frac{1}{5}} \tan t \, dt &= \cos^2 \frac{1}{5} \cdot t \Big|_{-\frac{1}{5}}^{\frac{1}{5}} + \sin^2 \frac{1}{5} (\tan t - t) \Big|_{-\frac{1}{5}}^{\frac{1}{5}} \\ &= \frac{2}{5} \cdot \cos^2 \frac{1}{5} + 2 \cdot \sin^2 \frac{1}{5} \cdot \left(\tan \frac{1}{5} - \frac{1}{5} \right) \\ &= \frac{2}{5} \left(\cos^2 \frac{1}{5} - \sin^2 \frac{1}{5} \right) + 2 \sin^2 \frac{1}{5} \cdot \tan \frac{1}{5} = \frac{2}{5} \cdot \cos \frac{2}{5} + 2 \sin^2 \frac{1}{5} \cdot \tan \frac{1}{5} \end{aligned}$$

Note by editor:

Many thanks to Florică Anastase-Romania for typed solutions.