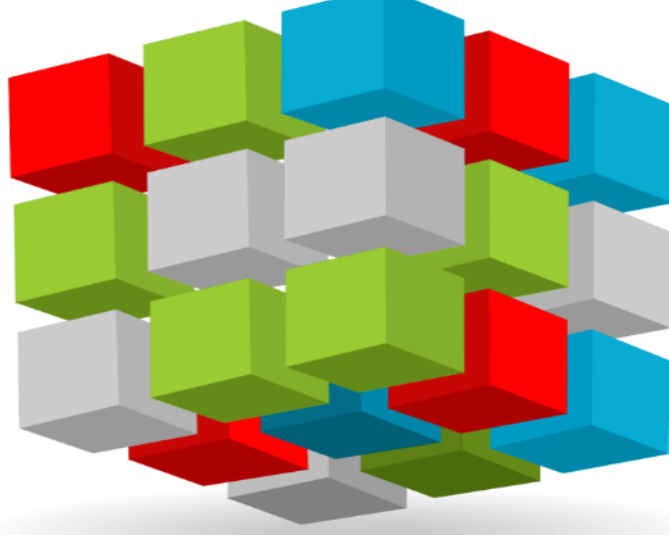


# R M M

ROMANIAN MATHEMATICAL MAGAZINE

www.ssmrmh.ro



In any  $\Delta ABC$  the following relationship holds:

$$2(\cos(A - B) + \cos(B - C) + \cos(C - A)) \leq 3 \sum_{cyc} \frac{\sin \frac{A}{2}}{\cos \frac{B}{2} \cos \frac{C}{2}}$$

When equality holds?

Proposed by Nguyen Van Canh-Vietnam

Solution by Marian Ursărescu-Romania

$$\sum \frac{\sin \frac{A}{2}}{\cos \frac{B}{2} \cos \frac{C}{2}} = \sum \frac{\sin \frac{A}{2} \cos \frac{A}{2}}{\cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}} = \frac{1}{2} \sum \frac{\sin A}{\cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}} \quad (1)$$

$$\text{But in any } \Delta ABC: \sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} \quad (2)$$

$$\text{From (1)+(2)} \Rightarrow \sum \frac{\sin \frac{A}{2}}{\cos \frac{B}{2} \cos \frac{C}{2}} = 2 \Rightarrow \text{we must show:}$$

$$\cos(A - B) + \cos(B - C) + \cos(C - A) \leq 3, \text{ which it is true, because}$$

$$\cos(A - B) \leq 1, \cos(B - C) \leq 1$$

$$\text{and } \cos(C - A) \leq 1, \text{ equality holds for } A = B = C.$$

Note by editor:

Many thanks to Florică Anastase-Romania for typed solution.