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If $x > 0$ then:

$$x^2 - 3x + 1 \geq logx^x - x^{x+1}$$

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Solution by Michael Sterghiou-Greece

$$x^2 - 3x + 1 \geq logx^x - x^{x+1} \quad (1)$$

$x^2 + 1 \overset{Am-Gm}{\geq} 2x$ *so (1) becomes the stronger inequality*

$-x - xlogx + x^{x+1} \geq 0$ *or* $x > 0 \Rightarrow -1 - logx + x^x \geq 0$

Consider the function $f(x) = x^x - logx - 1, f'(x) = x^x(1 + logx) - \frac{1}{x}$

If $x < 1$ *then* $x^x < 1; 1 + logx < 1; \frac{1}{x} > 1 \Rightarrow f'(x) < 0$

If $x > 1$ *then* $f'(x) > 0, f'(1) = 0$.

Therefore $f(x) \uparrow, x \in [1, \infty); f(x) \downarrow x \in (0, 1]$ *hence:* $f(1) = 0$ *is a global min on* $(0, \infty)$

because $\lim_{x \to 0_+} f(x) = +\infty; \lim_{x \to \infty} f(x) = +\infty$.

Now: $f(x) \geq f(0) = 0$. *Done!*

Note by editor:

Many thanks to Florică Anastase-Romania for typed solution.