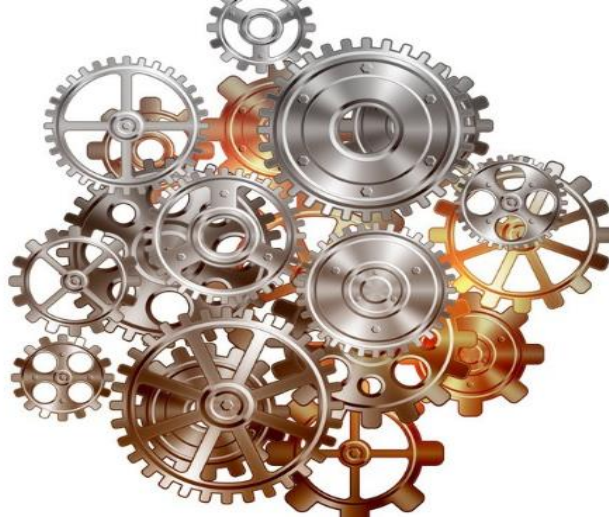


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Solve for real numbers:

$$\begin{cases} xy + yz + zx = 26 \\ \frac{48 + yz(y + z)}{(x - y)(x - z)} + \frac{48 + zx(z + x)}{(y - x)(y - z)} + \frac{48 + xy(x + y)}{(z - x)(z - y)} = 9 \\ xyz = 24 \end{cases}$$

Proposed by Daniel Sitaru-Romania

Solution by Tran Hong-Dong Thap-Vietnam

$$x \neq y; y \neq z; z \neq x, x, y, z \neq 0$$

$$xy + yz + zx = 26; xyz = 24$$

$$\begin{aligned} \frac{48 + yz(y + z)}{(x - y)(x - z)} + \frac{48 + zx(z + x)}{(y - x)(y - z)} + \frac{48 + xy(x + y)}{(z - x)(z - y)} = 9 &\Leftrightarrow \\ 48 \left(\frac{1}{(x - y)(x - z)} + \frac{1}{(y - x)(y - z)} + \frac{1}{(z - x)(z - y)} \right) + & \\ + \left(\frac{yz(y + z)}{(x - y)(x - z)} + \frac{zx(z + x)}{(y - x)(y - z)} + \frac{xy(x + y)}{(z - x)(z - y)} \right) = 9 &\Leftrightarrow \\ 48 \left(\frac{-(y - z) - (z - x) - (x - y)}{(x - y)(y - z)(z - x)} \right) + & \\ + \left(\frac{yz(y + z)}{(x - y)(x - z)} + \frac{zx(z + x)}{(y - x)(y - z)} + \frac{xy(x + y)}{(z - x)(z - y)} \right) = 9 & \end{aligned}$$

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$$\left(\frac{-yz(y+z)(y-z) - zx(z+x)(z-x) - xy(x+y)(x-y)}{(x-y)(y-z)(z-x)} \right) = 9$$

$$yz(y^2 - z^2) + zx(z^2 - x^2) + xy(x^2 - y^2) = -9(x-y)(y-z)(z-x)$$
$$x^3y + y^3z + z^3x - (xy^3 + yz^3 + zx^3) + 9(x^2z + z^2y + y^2x - xz^2 - zy^2 - yx^2) = 0$$

$$(x-y)(y-z)(z-x)(x+y+z-9) = 0 \stackrel{x \neq y; y \neq z; z \neq x}{\iff} x+y+z-9 = 0$$

$$x+y+z = 9$$

So, by Vieta's Theorem:

$$X^3 - 9X^2 + 26X - 24 = 0$$

$$(x; y; z) = (2; 3; 4) \text{ and cyclic.}$$

Note by Editor:

Many thanks to Florică Anastase-Romania for typed solution.