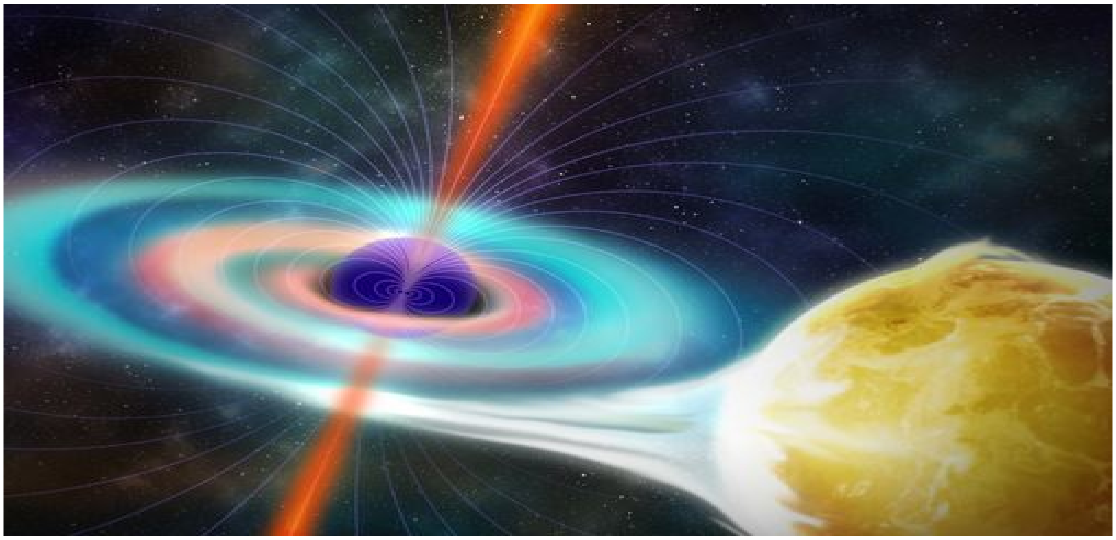
R M M

ROMANIAN MATHEMATICAL MAGAZINE
www.ssmrmh.ro



UP.278 If $a, b \in \mathbb{R}$ then:

$$\int_a^b \int_a^b (\cos x \cos y \cos(x+y))\,dx\,dy + \frac{1}{8}(b-a)^2 \geq 0$$

Proposed by Daniel Sitaru – Romania

Solution by proposer:

We start from:

$$\left(\cos x + \frac{1}{2}(\cos(x+2y))\right)^2 + \frac{1}{4}\sin^2(x+2y) \geq 0$$

$$\cos^2 x + \cos x \cos(x+2y) + \frac{1}{4}\cos^2(x+2y) + \frac{1}{4}\sin^2(x+2y) \geq 0$$

$$\cos^2 x + \cos x \cos(x+2y) \geq -\frac{1}{4}$$

$$2\cos^2 x + 2\cos x \cos(x+2y) \geq -\frac{1}{2}$$

$$2\cos^2 x + \cos 2y + \cos(2x+2y) \geq -\frac{1}{2}$$

$$2\cos^2 x - 1 + \cos^2 y + \cos(2x+2y) \geq -\frac{3}{2}$$

$$\cos 2x + \cos 2y + \cos(2x+2y) \geq -\frac{3}{2}$$

$$2\cos(x+y)\cos(x-y)+2\cos^2(x+y)-1\geq -\frac{3}{2}$$

$$2\cos(x+y)\left(\cos(x-y)+\cos(x+y)\right)\geq -\frac{1}{2}$$

$$\cos(x+y)\cdot 2\cos x\cos y\geq -\frac{1}{4}$$

$$8\cos x\cos y\cos(x+y)+1\geq 0$$

$$\cos x\cos y\cos(x+y)+\frac{1}{8}\geq 0$$

$$\int_a^b\int_a^b \cos x\cos y\cos(x+y)\,dx\,dy+\frac{1}{8}(b-a)^2\geq 0$$